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EUCLID
FOR BEGINNERS
BOOKS I. AND II.

F. B. HARVEY, M. A.

E U C L I D

BOOKS I. & II.

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EUCLID

FOR BEGINNERS

BOOKS I. AND II.

WITH SIMPLE EXERCISES

BY THE

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182

111

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THERE are already so many and such excellent editions of Euclid that to offer another, though only of Books I. and II., may be considered both superfluous and presuming. Yet, it would be somewhat rash, on the other hand, to say that we have arrived at perfection in the matter, and that no further attempts at improvement in the publication of Euclid—for beginners, at least—need be made.

Every teacher of Euclid knows, and a long experience has confirmed the knowledge in myself, that the better the mere typical arrangement of the text, the more attractive the study of Euclid in itself is, and the quicker, and more complete, is the progress which the beginner makes. If, further, the text can be expressed more distinctly, and be brought more fully within the boy's comprehension, so that he can readily perceive what he really has to do in 'learning a Proposition,' as it is called; and be led to put a definite value upon the statement of the abstract truths he finds demonstrated, then every possible aid is afforded him in his often difficult and uninteresting work. These two points are aimed at in this edition of Euclid, as strictly a book for beginners. If the way can be smoothed thus far, the rest of the road is fully open.

As regards the first requisite, the typical arrangement of the text, it is hoped that the distinct expression, in red ink, of the particular enunciation with reference to the figure employed, and the special statement of the point or points to

be proved in the Proposition, will be found to contribute materially to the advantages spoken of. A corresponding improvement is sought in the further use, to Prop. XXVI. inclusive, of red ink, to denote the lines employed in the 'construction' of the several figures.

And with reference to the second requisite, the language of the text itself, it is believed that by a very simple deviation from the usual phraseology of the demonstration, without any sacrifice of geometrical or logical truth, a great help will be afforded to the scholar, both in learning and remembering the several Propositions.

The alteration is chiefly this :—Through the whole of the First Book there runs, as a thread, the frequent comparison of two triangles. This comparison is usually made in a manner which, to a boy, seems unnecessarily cumbrous and puzzling. Of the three parts to be taken in each of the two triangles, as equal to each other, each to each, two are first taken, and their respective equalities stated ; then the third in each is taken, and their equality asserted. By this a kind of break is made in the argument, which acts as a hindrance to a learner, unimportant as it may appear to be. Now, surely, to take the three parts in each triangle, and to compare them, each with each, once for all, is just as correct, geometrically and logically, and it is certainly by far the simpler plan. A reference to the proof of Proposition V. will explain my meaning, and show, I think, the advantage claimed.

This point allowed, the application of the principle through the whole Book tends greatly to simplify it, for the purposes stated. Again, another common defect in the text is that, after the necessary comparison of two triangles has been made, while often only one of the consequences deducible is required in the argument of the Proposition, all the consequences are stated. Surely this also is cumbrous and puzzling.

 *For these, and other reasons which will speak for them-*

selves in the several Propositions, I presume to issue this edition of Books I. and II. My original purpose would have been answered by the publication of Book I. only, but the addition of Book II., which has also its especial features, will make the whole available for some of the examinations which have to be undergone, especially since all symbols, or abbreviations, which the several examining bodies disallow, are carefully excluded. I do not suppose that I have found out the 'royal road.' I shall be more than satisfied if I do but indicate another step in that direction.

I have not thought it necessary to introduce additional Problems, or Riders. In doing so I should be departing somewhat from the object I have chiefly in view, of preparing a book especially suitable to beginners, who, when they are sufficiently advanced for such work, will find excellent and ample material for it elsewhere. I have, however, appended some very simple Exercises, generally variations authorised by the Propositions under which they are placed. These, if they are not thought superfluous, will contribute to a more thorough understanding of the Propositions themselves, and help to train the scholar for the higher efforts of the kind which he may afterwards have to make.

Any favourable testimony that the use of this book may warrant will be appreciated, and suggestions and even hostile criticism shall have a hearty welcome, and the fullest attention.

F. B. HARVEY.

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INTRODUCTION.

THE PROPOSITIONS of Euclid are divided into two kinds, Problems and Theorems. Problems give something to be done, as the making of a Triangle. Theorems state something to be proved, as that the angles at the base of an Isosceles Triangle are equal to each other. But in Problems as well as in Theorems, argument is employed. In a Theorem the necessity of argument is apparent. In a Problem, after we have done what is required, we have to prove the accuracy of our work. Every Proposition, therefore, is to be considered as a process of reasoning. This process is carried on step by step. We start from known truths or suppositions admitted, through others, plainly flowing from or connected with them, till we arrive at the conclusion to which they unavoidably lead us. Mere assertion is allowed no place in this process. The truth of everything stated must be capable of, and have, its necessary proof. This fact the scholar must bear in mind. He must remember that to *learn a Proposition* is to get up an argument by which the statement contained in that Proposition is established. To *say*, or to write out, a *Proposition* is to reproduce that argument complete. When the scholar has grasped this idea of a Proposition, he will have made a great step, not only towards the study of Euclid successfully, but also attractively: 'a consummation devoutly to be wished.'

From this general view of a Proposition, in Euclid, we can now proceed to a closer one.

Every Proposition contains what is called, 1. The Enunciation. 2. The Construction. 3. The Demonstration, or Proof.

1. The Enunciation is the original statement of the Problem, or Theorem, and it is of two kinds, general and particular.

The General Enunciation is the statement asserted generally, and printed here, and usually, in *Italics*.

The Particular Enunciation is the repetition of the General, asserted with reference to the particular case in which we are about to consider it. This gives us also, in distinct terms, the statement to be proved, which is printed here in red type.

2. The Construction is the addition to the lines or figures, originally given, of such other lines, or figures, as are necessary to the argument required. These Construction lines and figures are also printed here, as far as Prop. XXVI. inclusive, in red.

3. The Demonstration, or Proof, follows on the principles, above explained, of starting from truths known, or suppositions admitted, through others flowing from, or connected with, them—such as Definitions, Axioms, Postulates, Hypotheses, and other and *previous* Propositions—till we reach the evident conclusion, and this is the statement required in the Particular Enunciation to be proved. This conclusion is printed here in red type, which is thus used to show that the argument, or process of reasoning, employed is only an intervening part of the entire Proposition, and that the statement, originally made, has been demonstrated, as required.

E U C L I D.

BOOK I.



DEFINITIONS.

1.

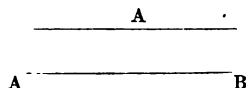
A POINT is that which has position, but not magnitude.

A geometrical point cannot be represented without magnitude. Its *position* is denoted by a letter, as the point A.

2.

A LINE is length without breadth.

A geometrical line cannot be represented without breadth. It is denoted by a letter, or letters, placed on the line, or at its extremities.



3.

THE EXTREMITIES of a line are points.

4.

A STRAIGHT LINE is that which lies evenly between its extreme points.

This is sometimes called a *right line*.

5.

A SUPERFICIES is that which has only length and breadth.

A geometrical superficies, or, as it is sometimes called, *surface*, cannot be represented without depth, or thickness. The shadow of

any object gives us the best idea of a superficies, or surface. A superficies is denoted by letters placed at its sides, or extremities.

6.

THE EXTREMITIES OF A SUPERFICIES are lines.

7.

A PLANE SUPERFICIES is that in which any two points being taken, the straight line between them lies wholly in that superficies.

A Plane Superficies is sometimes called 'A Plane.'

A brick has six plane superficies, or surfaces.

8.

A PLANE ANGLE is the inclination of *two lines* to each other in a Plane, which meet together, but are not in the same direction.

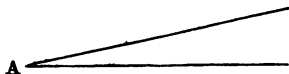
The term 'angle' in this definition denotes the *opening* which exists between two lines meeting in a point, in a plane. These 'two lines' may be straight, or curved, either or both. The only restriction is that they must meet each other, not in the same direction, in a Plane. These *Plane Angles* are not introduced in Elementary Geometry, which refers entirely to the *Plane Rectilineal Angle* spoken of in Definition 9.

9.

A PLANE RECTILINEAL ANGLE is the inclination of *two straight lines* to each other, which meet together, but are not in the same straight line.

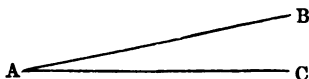
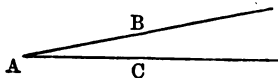
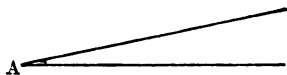
This definition is a very important one, and it must be distinctly understood.

a. The plane rectilineal angle—*angulus, a corner*—is simply the opening between two straight lines meeting at, or starting from, the same point, as A.



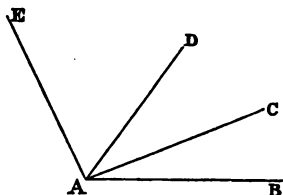
b. The point where these lines meet is called the *vertex*, and the lines themselves the *arms* of the angle.

c. An angle is denoted by a letter placed at its vertex, or by this letter with two others placed on, or at the extremities of, the lines or arms of the angle.



Thus we have the angle A, or the angle BAC. The vertex letter is always the *middle* one of the three. Hence the angle BAC is the same as the angle CAB.

d. When several lines meet and form several angles, at the same vertex, we may consider one of such angles as a part of two or more; and we may consider two or more of such angles combined as one angle.

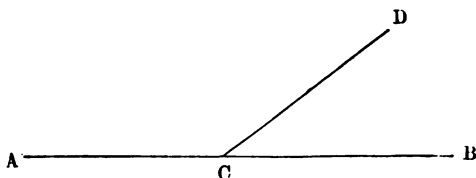


Thus we may consider the angle BAC as a part of the angle BAD, or BAE; and we may consider the angles BAC and CAD and DAE combined as one angle BAE, &c.

e. The magnitude of an angle is explained under def. 15.

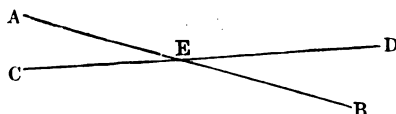
f. When a straight line meets another straight line at a point.

in the latter which is not one of its extremities, the angles thus formed are *adjacent* angles.



Thus the angles ACD and BCD are adjacent angles.

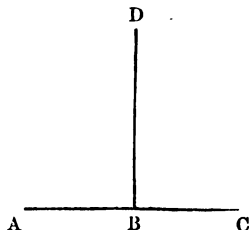
g. When two straight lines cut or intersect each other, the four angles thus formed are pairs of *vertically opposite* angles.



Thus the angles AEC and BED are vertically opposite angles ;
as also are the angles AED and BEC.

10.

When a straight line standing upon another straight line makes the adjacent angles equal to each other, each of these angles is called a **RIGHT ANGLE**, and *each* straight line is said to be **PERPENDICULAR** to the other.



Thus each of the angles ABD and CBD is a right angle ; DB is perpendicular to AC, or to AB and BC ; also AB and CB are each perpendicular to BD.

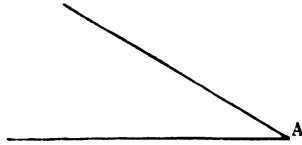
11.

AN OBTUSE ANGLE is one which is greater than a right angle.



12.

AN ACUTE ANGLE is one which is less than a right angle.



13.

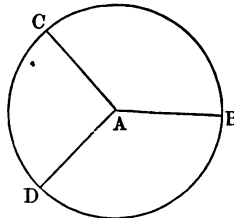
A TERM OR BOUNDARY is the extremity of anything.

14.

A FIGURE is that which is contained by one or more boundaries.

15.

A CIRCLE is a plane figure contained by one line called the CIRCUMFERENCE, and is such that all straight lines drawn from a certain point within it, called the CENTRE, to the circumference are equal to each other. Each of such equal straight lines is called a RADIUS.

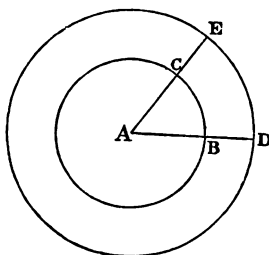


Thus BCD is a *circle* with *circumference* BCD, the centre A,

and each of the lines AB, AC, and AD is a *radius*. A part of the circumference, as BC, is called an *arc*.

Magnitude of an Angle.

1. The entire circumference of *every* circle is divided into 360 parts, called *degrees*, each deg. = 60 minutes, and each min. = 60 seconds; and an angle is said to contain as many degrees°, minutes', and seconds'', as are contained in the *arc*, or that part of the circumference which lies between the two lines forming the angle; the angular point, or *vertex*, being the *centre* of the circle.



Thus the angle BAC contains as many deg., min., and secs., as the arc BC in the smaller circle.

The angle DAE contains as many deg., min., and secs. as the arc DE in the larger circle.

Now, it can be proved that arc BC is the *same part* of its circumference that DE is of its circumference.

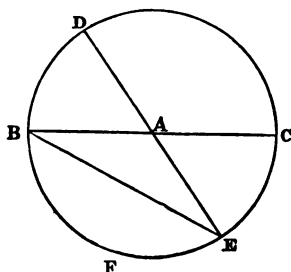
Therefore the angle BAC = the angle DAE, and *hence*—

2. The length of the arms of an angle makes no difference in the magnitude of that angle.

Note, also, the arms need not be of the same length. For it is plain that the angle BAC = the angle BAE, &c.

3. The arms of a Right Angle include *one-fourth* part of the circumference. A *Right Angle* contains, therefore, 90 degrees; an *obtuse* angle contains *more*, and an *acute* angle *less*, than 90 degrees.

through the *centre*, and terminated both ways by the circumference.



Thus BAC and DAE are diameters. A straight line drawn in a circle, *not through the centre*, and terminated both ways by the circumference is called a *Chord*, as the straight line BE.

17.

A **SEMICIRCLE** is that part of the circle which is contained by a diameter and the arc it cuts off.

In the above figure, CDB, BEC, and ECD are Semicircles.

18.

A **SEGMENT OF A CIRCLE** is that part of the circle which is contained by a chord and its arc.

In the above figure the chord BE divides the circle into two segments BDCE and BFE.

19.

RECTILINEAL FIGURES are those which are contained by right or straight lines.

20.

A **TRIANGLE** is contained by three straight lines.

21.

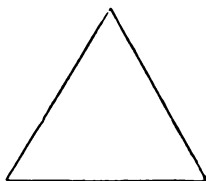
QUADRILATERAL FIGURES are contained by four straight lines.

22.

MULTILATERAL FIGURES, or POLYGONS, are contained by more than four straight lines.

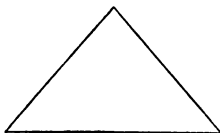
23.

AN EQUILATERAL TRIANGLE has all its sides equal to each other.



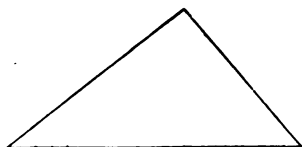
24.

AN ISOSCELES TRIANGLE has two of its sides equal to each other.



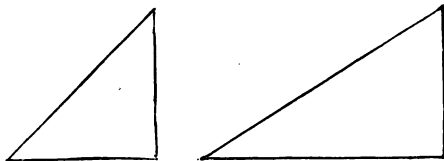
25.

A SCALENE TRIANGLE has all its sides unequal to each other.



26.

A RIGHT-ANGLED TRIANGLE has one right angle.

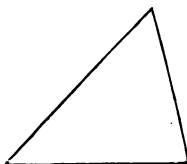


27.

AN OBTUSE-ANGLED TRIANGLE has one obtuse angle.

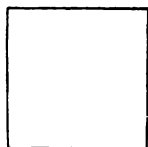
28.

AN ACUTE-ANGLED TRIANGLE has *three* acute angles.



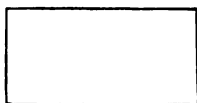
29.

A SQUARE is a Quadrilateral having all its sides equal, and all its angles right angles.



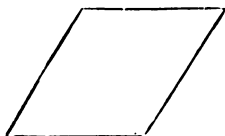
30.

AN OBLONG is a Quadrilateral which has not all its sides equal, but all its angles are right angles.



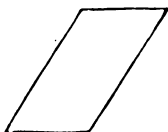
31.

A RHOMBUS is a Quadrilateral having all its sides equal, but its angles are not right angles.



32.

A RHOMBOID is a Quadrilateral having its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.



33.

All other Quadrilaterals besides these are called TRAPEZIUMS.

Some writers on Mensuration of Surfaces speak of a *Trapezoid* as a Quadrilateral having *one* pair of opposite sides parallel, and consider the *Trapezium* as having neither pair of opposite sides parallel. This is not in accordance with Euclid's language in Book I. Prop. 35, where a Quadrilateral with one pair of opposite sides parallel is called a Trapezium.

This thirty-third def. is limited by Def. 34, which is usually appended as a Note to the Enunciation of Prop. 34, Book I.

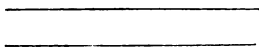
34.

A PARALLELOGRAM is a Quadrilateral of which the *opposite sides* are parallel ; and the *diagonal*, or diameter, is the straight line joining two of its opposite angles.

The Square, Oblong, Rhombus, and Rhomboid are each of them Parallelograms, as this definition shows. For the terms Rhombus and Rhomboid that of Parallelogram is often used ; and for Oblong the term Rectangle.

35.

PARALLEL STRAIGHT LINES are such as are in the same plane, and which, being continually produced, never meet.



POSTULATES.

1.

Let it be granted that a straight line may be drawn from any one point to any other point.

2.

That a terminated straight line may be produced to any length in a straight line.

3.

That a circle may be described from any centre, at any distance from that centre.

The Postulates are 'Requests' that Euclid makes for certain things to be allowed as permissible in the study of Geometry. They are but three: 1. The *drawing* of a straight line from any one point to any other. 2. The *producing*, to any length, of a straight line already drawn. 3. The describing of a circle from *any centre* with *any radius*.

AXIOMS.

1.

Things which are equal to the same thing are equal to one another.

2.

If equals be added to equals, the wholes are equal.

3.

If equals be taken from equals, the remainders are equal.

4.

If equals be added to unequals, the wholes are unequal.

5.

If equals be taken from unequals, the remainders are unequal.

6.

Things which are double of the same are equal to one another.

7.

Things which are halves of the same are equal to one another.

8.

Magnitudes which coincide with one another—that is, which fill exactly the same space—are equal to one another.

9.

The whole is greater than its part.

10.

Two straight lines cannot enclose space.

11.

All right angles are equal to one another.

12.

If a straight line meets two straight lines so as to make the two interior angles on the same side of it, taken together, less than two right angles, these straight lines being continually produced shall at length meet upon that side on which are the angles which are less than two right angles.

The Axioms are 'Common Notions,' or self-evident Truths. To them Euclid, on this ground, claims assent. Axioms 10, 11, and 12 are considered by some to be of the nature of Postulates rather than Axioms. This, however, is a distinction the consideration of which the beginner in Euclid may postpone.

HINTS TO THE LEARNER.

1. Make the figures of a good size, and as accurately as possible. A well-drawn figure is of great value towards the understanding of the Proposition.

2. Do not *copy* the figures of any Proposition, but draw them, step by step, as directed in the 'Construction.'

3. Remember that in the 'Proof' of any Proposition Euclid employs those Propositions only which are *previous* to the one under consideration. He never expects you to have a knowledge beyond what you ought thus to have already acquired.

EXPLANATION OF TERMS.

A COROLLARY is a Theorem, or Problem, which arises easily and directly from the Proposition to which it is attached.

HYPOTHESIS is a *supposition* assumed, for the time, to be true.

Q. E. F. stand for *Quod erat faciendum*, meaning *which was to be done*. They stand at the end of *Problems*.

Q. E. D. stand for *Quod erat demonstrandum*, meaning *which was to be demonstrated* or proved. They stand at the end of *Theorems*.

The following abbreviations are used :

<i>ax.</i> axiom.	<i>ext.</i> exterior.
<i>alt.</i> alternate.	<i>fig.</i> figure.
<i>comp.</i> complement.	<i>hyp.</i> hypothesis.
<i>cons.</i> construction.	<i>int.</i> interior.
<i>cor.</i> corollary.	<i>opp.</i> opposite.
<i>def.</i> definition.	<i>post.</i> postulate.



EUCLID.

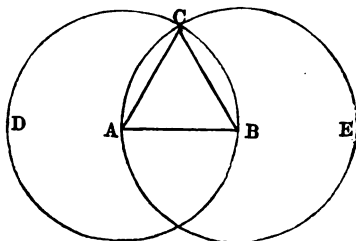
BOOK I.

PROP. I. PROBLEM.

To describe an equilateral triangle upon a given finite straight line.

Let AB be the given straight line.

It is required to describe on AB an equilateral triangle.



CONSTRUCTION.—1. From centre A , with distance AB , describe the circle BCD (post. 3).

2. From centre B , with distance BA , describe the circle ACE .

3. From point C , where the circles cut each other, draw CA and CB (post. 1).

Then, it is to be proved that

ABC is an equilateral triangle described upon AB .

PROOF.—*Because* A is the centre of the circle BCD , *therefore* $AB=AC$ (def. 15).

Similarly, *because* B is the centre of the circle ACE , *therefore* $BA=BC$.

But it has been proved that $BA=AC$, *therefore* $AC=BC$ (ax. 1), *and therefore* AB , BC , and CA = each other.

Therefore, it is proved, as required, that

ABC is an equilateral triangle, described upon AB .

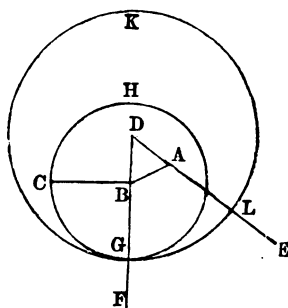
Q. E. F.

PROP. II. PROBLEM.

From a given point to draw a straight line equal to a given straight line.

Let A be the given point, and BC the given straight line.

It is required to draw from A a straight line equal to BC.



CONSTRUCTION.—1. From A to B draw the straight line AB (post. 1).

2. Upon AB describe the equilateral triangle BDA (I. 1).

3. Produce DA and DB to the points E and F (post. 2).

4. From centre B, with distance BC, describe the circle CHG, cutting DF in G (post. 3).

5. From centre D, with distance DG, describe the circle GKL cutting DE in L.

Then, it is to be proved that

AL is the line drawn from A = BC.

PROOF.—*Because* B is the centre of the circle CHG, *therefore* BG = BC (def. 15).

Similarly, *because* D is the centre of the circle GKL, *therefore* DG = DL.

But in the lines DG and DL we have DB = DA (cons.), *therefore* BG = AL (ax. 3).

Also it has been shown that BG = BC; *therefore* AL = BC (ax. 1).

Therefore, it is proved, as required, that

AL is the line drawn from A = BC.

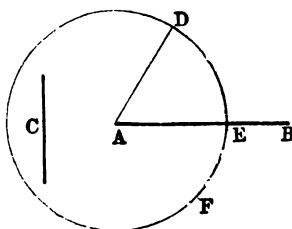
Q. E. F.

PROP. III. PROBLEM.

From the greater of two given straight lines to cut off a part equal to the less.

Let AB and C be the two given straight lines, of which AB is the greater.

It is required to cut off from AB , the greater, a part equal to C , the less.



CONSTRUCTION.—1. From A draw $AD = C$ (I. 2).

2. From centre A , with distance AD , describe the circle DEF , cutting AB in E (post. 3).

Then, it is to be proved that

AE is cut off from $AB = C$.

PROOF.—Because A is the centre of the circle DEF , therefore $AE = AD$ (def. 15).

But $AD = C$ (cons.); therefore $AE = C$ (ax. 1).

Therefore, it is proved, as required, that

AE is cut off from $AB = C$.

Q. E. F.

Exercises.

1. Describe an equilateral triangle on a given straight line MN , with the vertex, O , below MN .

2. Prove Prop. II. when A is joined to C , instead of to B .

N.B.—The Exercises given in the following pages are not necessarily connected with the Proposition under which they are placed. But they are strictly confined to that or to previous Propositions.

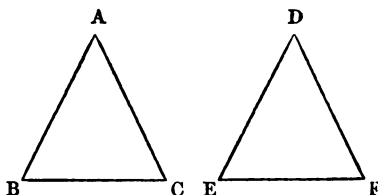
PROP. IV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, and have also the angles contained by those sides equal to each other, then they shall have their bases, or third sides, equal; and the two triangles shall be equal; and their other angles shall be equal, each to each, viz., those to which the equal sides are opposite.

In the triangles ABC and DEF let the sides AB and AC, and their angle BAC, in the former = the sides DE and DF, and their angle EDF, in the latter, each to each.

Then it is to be proved that

1. The base BC = the base EF.
2. The triangle ABC = the triangle DEF.
3. The angle ABC = the angle DEF.
4. The angle ACB = the angle DFE.



PROOF.—If the triangle ABC be placed upon the triangle DEF so that the point A is on the point D, and the side AB on the side DE, then, *because* $AB=DE$ (hyp.), *therefore* the point B shall coincide with the point E, and the side AC shall coincide with the side DF.

Next, *because* the side AB coincides with the side DE, and *because* the angle BAC = the angle EDF (hyp.), *therefore* the side AC shall fall on the side DF, and, *because* the point A coincides with the point D, and $AC=DF$, (hyp.), *therefore* the point C shall coincide with the point F.

But we have also seen that the point B coincides with the point E; *therefore* the whole base BC shall coincide with the whole base EF.

For, if the point B coincides with the point E, *and* the point C coincides with the point F, *then, if* the whole base BC does *not* coincide with the whole base EF, we have two straight lines enclosing a space, which is impossible (ax. 10).

Therefore,

1. The base BC coincides with the base EF.
2. The triangle ABC coincides with the triangle DEF.
3. The angle ABC coincides with the angle DEF.
4. The angle ACB coincides with the angle DFE.

And therefore, it is proved (ax. 8), as required, that

1. The base BC = the base EF.
2. The triangle ABC = the triangle DEF.
3. The angle ABC = the angle DEF.
4. The angle ACB = the angle DFE.

Wherefore,

If two triangles, &c.

Q. E. D.

Exercises.

1. Given the straight lines AB and CD, of which AB is the greater; it is required to produce CD to make it = AB.
2. Prove Prop. IV. when the triangle DEF is applied to the triangle ABC.

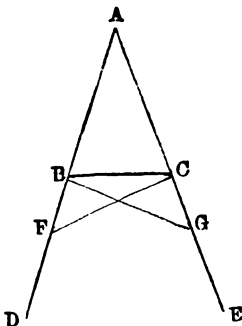
PROP. V. THEOREM.

The angles at the base of an isosceles triangle are equal to each other, and, if the equal sides are produced, the angles upon the other side of the base are also equal to each other.

Let ABC be an isosceles triangle with the side $AB =$ the side AC , and let AB and AC be produced to D and E respectively.

Then it is to be proved that

1. The angles ABC and ACB , at the base $=$ each other.
2. The angles DBC and ECB , upon the other side of the base, also $=$ each other.



CONSTRUCTION.—1. In BD take any point F .

2. From AE cut off $AG = AF$ (I. 3), and join BG and CF .

PROOF.—*Because* in the triangles FAC and GAB we have the sides FA and AC , and their angle FAC , in the former $=$ the sides GA and AB , and their angle GAB , in the latter, each to each (hyp. and cons.), *therefore* the base $CF =$ the base BG , the angle $ACF =$ the angle ABG , and the angle $AFC =$ the angle AGB (I. 4), *i.e.* the angle $BFC =$ the angle CGB (note 2 def. 15).

Next, *because* in the triangles BCF and CBG we have

the sides BF and FC, and their angle BFC, in the former = the sides CG and GB, and their angle CGB, in the latter, each to each (cons. and proof above), *therefore* the angle BCF = the angle CBG, and the angle FBC = the angle GCB (I. 4), *i.e.* the angle DBC = the angle ECB (note 2 def. 15), *and these are the angles upon the other side of the base.*

Further, *because* the angle ABG = the angle ACF, *and* the angle CBG = the angle BCF, as already proved, *therefore* if the angle CBG be taken from the angle ABG, *and* the angle BCF be taken from the angle ACF, *then* the remaining angle ABC = the remaining angle ACB (ax. 3), *and these are the angles at the base.*

Therefore, it is proved, as required, that

1. The angles ABC and ACB, at the base = each other, and
2. The angles DBC and ECB, upon the other side of the base, also = each other.

Wherefore,

The angles at the base of an isosceles triangle, &c.

Q. E. D.

Cor. Every equilateral triangle is also equiangular.

NOTE.—It may assist the scholar, in learning this proposition, to observe that in the *first* step in the proof the *larger* pair of triangles, FAC and GAB, and in the *second* step the *smaller* pair, BCF and CBG, are taken.

He will also notice that the equality of the angles ‘on the other side of the base’ is proved, before the equality of those ‘at the base’ is demonstrated.

Exercise.

Given an isosceles triangle BAC with the vertical angle at A bisected by AD drawn to BC; *prove* that AD is perpendicular to BC.

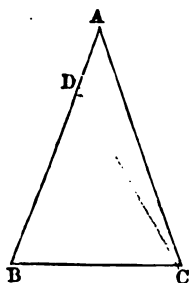
PROP. VI. THEOREM.

If two angles of a triangle are equal to each other, then the sides also which subtend, or are opposite to, the equal angles, are equal to each other.

Let ABC be a triangle having the angle $ABC =$ the angle ACB :

Then it is to be proved that

The side $AB =$ the side AC .



CONSTRUCTION.—Suppose that AB is *greater* than AC .

From AB cut off a part $DB = AC$ (I. 3) and join CD .

PROOF.—*Because* in the triangles DBC and ACB we have the sides DB and BC and their angle DBC , in the former $=$ the sides AC and CB , and their angle ACB , in the latter, each to each (hyp. and cons.), *therefore* the triangle $DBC =$ the triangle ACB (I. 4), *i.e.* a part $=$ the whole, which is absurd (ax. 9).

Therefore, the supposition that AB is *greater* than AC is absurd.

Similarly the supposition that AB is *less* than AC might be shown to be absurd.

Therefore, it is proved, as required, that

The side $AB =$ the side AC .

Wherefore,

If two angles of a triangle, &c.

Q. E. D.

Cor.—Every equiangular triangle is also equilateral.

N.B.—The proof here given is an instance of what is called *Indirect Demonstration*. This means, the truth of the statement asserted is only proved by showing that absurdity follows from an argument based on the supposition that such statement is not true.

The scholar must be careful to notice that the term ‘absurd,’ which is found in this method of proof, does not apply to the *entire* demonstration, but only to the *hypothesis* on which it is based. The chain of argument beyond this, in these cases, is strictly correct.

Exercises.

1. Prove the above Proposition on the supposition that AC is greater than AB .
2. If a straight line DA be drawn at right angles to another straight line BC from its middle point D , *prove*, if BA and CA be joined, that $BA = CA$.

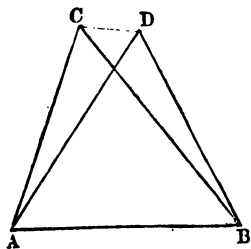
PROP. VII. THEOREM.

Upon the same base, and on the same side of it, there cannot be two triangles that have their sides terminated in one extremity of the base equal to each other, and likewise those which are terminated in the other extremity of the base equal to each other.

Suppose it possible that on the same base, AB , and on the same side of it—*e.g.* above it—there *can* be two triangles, ACB and ADB , having the sides AC and AD terminated in one extremity of the base, A , = each other, and having also the sides BC and BD terminated in the other extremity of the base, B , = each other ;

Then this supposition will present itself in three cases.

CASE I.—Where the vertex of each triangle falls *without* the other, as in the following figure.



CONSTRUCTION.—Join the vertices CD .

PROOF.—*Because* in the triangle ACD the side AC = the side AD (hyp.), *therefore* the angle ACD = the angle ADC (I. 5).

But the angle ACD is greater than the angle BCD (ax. 9); *therefore* the angle ADC is also greater than the angle BCD .

Again, *because* the angle BDC is greater than the angle ADC (ax. 9), *therefore* the angle BDC is *much greater* than the angle BCD .

Next, *because* in the triangle BDC the side BD = the side BC (hyp.), *therefore* the angle BDC = the angle BCD (I. 5).

But we have just proved that the angle BDC is "*much greater*" than the angle BCD .

Therefore, on the above supposition, the angle BDC is both equal to, and greater than, the angle BCD , which is absurd.

Therefore the above supposition is false as referred to CASE I., where the vertex of one triangle falls *without* the other.

We now pass on to consider this same supposition under

CASE II.—Where the vertex of one triangle, D , falls *within* the other, as in the following figure.

CONSTRUCTION.—1. Join the vertices CD.

2. Produce AC and AD to E and F respectively.

PROOF.—*Because* in the triangle ACD, the side AC = the side AD (hyp.) and these sides are produced to E and F respectively, *therefore* the angle ECD = the angle FDC (I. 5).

But the angle ECD is greater than the angle BCD (ax. 9); *therefore* the angle FDC is also greater than the angle BCD.

Again, *because* the angle BDC is greater than the angle FDC (ax. 9), *therefore* the angle BDC is *much greater* than the angle BCD.

Next, *because* in the triangle BDC, the side BD = the side BC (hyp.), *therefore* the angle BDC = the angle BCD (I. 5).

But we have just proved that the angle BDC is "*much greater*" than the angle BCD.

Therefore, on the above supposition, the angle BDC is *both equal to, and greater than*, the angle BCD, which is absurd.

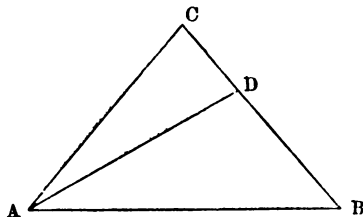
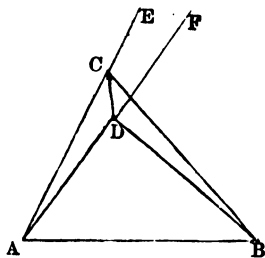
Therefore the above supposition is false as referred to CASE II., where the vertex of one triangle falls within the other.

CASE III.—Where the vertex of one triangle falls on a side of the other, as in the following figure.

In this case it is evident that if one pair of the sides which are terminated in one extremity of the base, e.g. AC and AD, terminated in A, equal each other, then the other pair, BC and BD, terminated in B, cannot be equal to each other, as the Proposition requires, and therefore this case is to be dismissed.

Wherefore,

Upon the same base, &c.



Q. E. D.

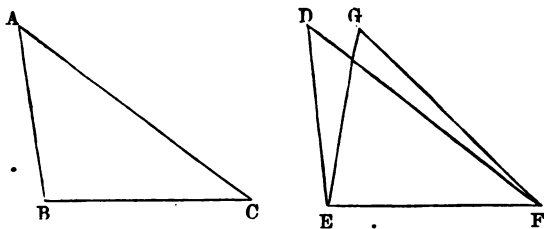
PROP. VIII. THEOREM.

If two triangles have the three sides of the one equal to the three sides of the other, each to each, then the angle which is contained by any two sides of the one triangle shall be equal to the angle contained by the two sides, equal to them, of the other.

In the triangles ABC and DEF, let the sides AB, BC, and CA in the former = the sides DE, EF, and FD in the latter, each to each.

Then it is to be proved that

1. The angle BAC = the angle EDF.
2. The angle ABC = the angle DEF.
3. The angle ACB = the angle DFE.



PROOF.—1. If the triangle ABC be placed upon the triangle DEF, so that the point B is on the point E, and the side BC upon the side EF, then, *because* $BC = EF$ (hyp.), *therefore* the point C will coincide with the point F, and *because* the point B coincides with the point E, and the point C with the point F, *therefore* the side BC coincides with the side EF.

Next, *because* the side BC coincides with the side EF *therefore* the sides BA and AC shall coincide with the sides ED and DF respectively.

For, if BC coincides with EF, and then BA and AC do *not* coincide with ED and DF, *suppose* that BA and AC have another direction, as EG and GF, then, *if this be true*, we shall have upon the same base EF, and upon the same side of it, two triangles in a manner which is impossible (I. 7).

Therefore, if BC coincides with EF, then BA and AC *must* coincide with ED and DF, *and* the angle BAC will coincide with, and equal, the angle EDF (ax. 8).

Therefore, it is proved, as required, that

1. The angle BAC = the angle EDF.

Similarly,

2. The angle ABC = the angle DEF.
3. The angle ACB = the angle DFE.

Wherefore,

If two triangles, &c.

Q. E. D.

N.B.—The equality of the two triangles, in every respect, follows from this Proposition, as it does from Prop. IV.

Exercises.

1. In the fig. Prop. I. if E be the point where the circles intersect below AB, and AE and BE be joined; *prove* that the angle ACB = the angle AEB.

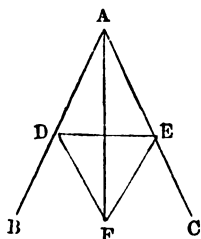
2. If BC be the base of an isosceles triangle with vertical angle at A; *prove* that if a line AD bisects the base BC, it bisects also the vertical angle BAC.

PROP. IX. PROBLEM.

To bisect a given rectilinear angle, i.e. to divide it into two equal parts.

Let BAC be the given rectilinear angle :

It is required to bisect it.



- CONSTRUCTION.—1. In AB take any point, D.
 2. From AC cut off $AE = AD$ (I. 3).
 3. On DE construct an equilateral triangle DEF (I. 1).
 4. Join AF.

Then it is to be proved that

The rectilinear angle BAC is bisected by the line AF.

PROOF.—*Because* in the two triangles DAF and EAF, we have the three sides DA, AF, and FD in the former = the three sides EA, AF, and FE, in the latter, each to each (cons.), *therefore* the angle DAF = the angle EAF (I. 8), i.e. the angle BAF = the angle CAF (note 2 def. 15).

Therefore, it is proved, as required, that

The rectilinear angle BAC is bisected by the line AF.

Q. E. F.

Exercise.

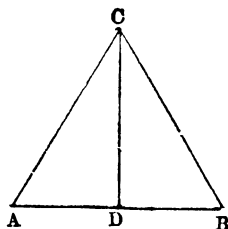
In the fig. Prop. I. if E be the point where the circles intersect below AB, and AE and BE be joined ; *prove* that AB bisects the angle CBE.

PROP. X. PROBLEM.

To bisect a given finite straight line, i.e. to divide it into two equal parts.

Let AB be the given finite straight line :

It is required to bisect it.



CONSTRUCTION.—1. On AB construct an equilateral triangle ABC (I. 1).

2. Bisect the angle ACB by CD , cutting AB in D (I. 9.)

Then it is to be proved that

The given straight line AB is bisected in D .

PROOF.—*Because* in the triangles ACD and BCD we have the sides AC and BC , and their angle ACD , in the former = the sides BC and CD , and their angle BCD , in the latter, each to each (cons.), *therefore* the base AD = the base BD (I. 4).

Therefore, it is proved, as required, that

The given straight line AB is bisected in D .

Q. E. F.

Exercise.

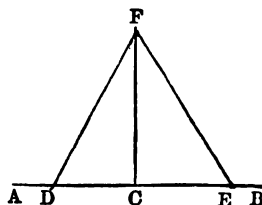
Given an isosceles triangle BAC with the vertical angle at A bisected by AD , meeting BC in D ; *prove* that BC is bisected in D .

PROP. XI. PROBLEM.

To draw a straight line at right angles to a given straight line, from a given point in the same.

Let AB be the given straight line, and C the given point in it :

It is required to draw from C a straight line at right angles to AB.



CONSTRUCTION.—1. Take any point D in AC.

2. Make $CE = CD$ (I. 3).

3. Upon DE construct an equilateral triangle DFE (I. 1) ;

4. Join FC.

Then it is to be proved that

The straight line FC is drawn from C at right angles to AB.

PROOF.—*Because* in the triangles DCF and ECF we have the three sides DC, CF, and FD, in the former = the three sides EC, CF, and FE, in the latter, each to each (cons.), *therefore* the angle DCF = the angle ECF (I. 8), and these are *adjacent* angles, and *therefore* right angles (def. 10).

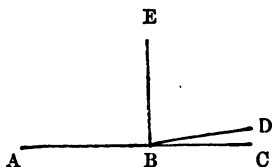
Therefore, it is proved, as required, that

The straight line FC is drawn from C at right angles to AB.

Q. E. F.

COROLLARY.

By help of this Problem it may be demonstrated that
Two straight lines cannot have a common segment.



Suppose it possible that ABC and ABD are two straight lines, with a common segment, or part, AB.

CONSTRUCTION.—From B draw BE at right angles to AB (I. 11).

PROOF.—*Because* ABC is a straight line (hyp.) with BE perp. to it (cons.), *therefore* the angle ABE = the angle EBC (def. 10); *and because* ABD is a straight line (hyp.) with BE perp. to it (cons.), *therefore* the angle ABE = the angle EBD (def. 10); *and therefore* the angle EBD = the angle EBC (ax. 1), *i.e.* the less = the greater, which is absurd (ax. 9).

The same result would follow for any other position of ABC and ABD, with AB as a part of each.

Therefore, it is proved, as required, that

Two straight lines cannot have a common segment.

Q. E. D.

Exercises.

1. Two points, A and B, are given *above* a straight line CD; *find* a point E in the straight line CD, so that if AE and BE be joined, $AE = BE$.

2. Find the same when A is *above*, and B *below*, the straight line CD, but the line joining A and B *not* perpendicular to CD.

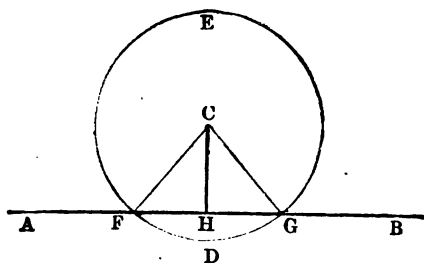
3. Find the same when A is *above*, and B *in*, the straight line CD.

PROP. XII. PROBLEM.

To draw a straight line at right angles to a given straight line of unlimited length, from a given point without it.

Let AB be the given straight line of unlimited length, or that which may be produced to any distance, both ways; and C the given point without it.

It is required to draw from C a straight line at right angles to AB .



CONSTRUCTION.—1. Take any point, D , upon the other side of AB .

2. From centre C with distance CD describe the circle EFG , cutting AB in F and G (post. 3).

3. Bisect FG in H (I. 10).

4. Join CF , CH , and CG .

Then it is to be proved that

The straight line CH is drawn from C at right angles to AB .

PROOF.—*Because* in the triangles FHC and GHC , we have the three sides FH , HC , and CF , in the former = the three sides GH , HC , and CG , in the latter, each to each

(cons.), *therefore* the angle $FHC =$ the angle GHC (I. 8); *and* these are *adjacent* angles, and *therefore* right angles (def. 10).

Therefore, it is proved, as required, that

The straight line CH is drawn from C at right angles to AB .

Q. E. F.

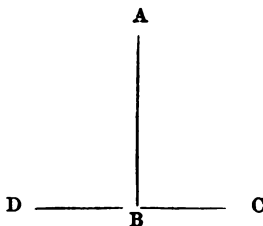
Exercises.

1. Let $ABCD$ be a square and AC and BD its diagonals; *prove* that $AC = BD$.
2. Let $ABCD$ be a rhomboid; *prove* that its *opposite* angles $=$ each other.
3. Let $ABCD$ be a rhombus with diagonal AC ; *prove* that the angle $DAC =$ the angle BAC , *and* that the angle $BCA =$ the angle DCA .

PROP. XIII. THEOREM.

The angles which one straight line makes with another upon one side of it are either two right angles, or are together equal to two right angles.

CASE I.—When the angles are equal to each other.



Let the straight line AB make with the straight line CD, on one side of it, the angles ABC and ABD = each other.

Then it is to be proved that

The angles ABC and ABD are two right angles.

PROOF.—*Because* the angle ABC = the angle ABD (hyp.), *therefore* each of them is a right angle (def. 10).

Therefore, it is proved, as required, that

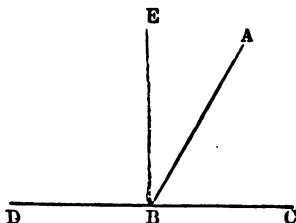
The angles ABC and ABD are two right angles.

CASE II.—When the angles are not equal to each other.

Let the straight line AB make with CD, on one side of it, the angles ABC and ABD not equal to each other.

Then it is to be proved that

The angles ABC and ABD are together = two right angles.



CONSTRUCTION.—Draw BE at right angles to CD (I. 11).

PROOF.—The angles DBE and CBE = two right angles (def. 10).

Now,¹ the angle EBC = the angles ABC and ABE , and, adding angle EBD to each of these, then the angles EBC and EBD = the angles ABC , ABE , and EBD (ax. 2).

Again,² the angle ABD = the angles ABE and EBD , and, adding angle ABC to each of these, then the angles ABD and ABC = the angles ABC , ABE , and EBD (ax. 2).

But, as above, the angles EBC and EBD = the same angles ABC , ABE , and EBD ; therefore the angles EBC and EBD = the angles ABD and ABC (ax. 1).

But the angles EBC and EBD = two right angles (cons.); therefore the angles ABD and ABC = two right angles (ax. 1).

Therefore, it is proved, as required, that

The angles ABC and ABD are together equal to two right angles.

Wherefore,

The angles which one straight line makes with another, &c.

Q. E. D.

¹ I.e. the double angle on the right side of the figure.

² I.e. the double angle on the left side of the figure.

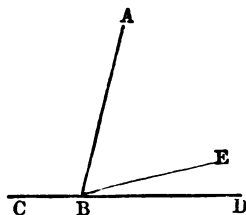
PROP. XIV. THEOREM.

If at a point in a straight line, two other straight lines, upon opposite sides of it, make the adjacent angles together equal to two right angles, then these two straight lines shall be in one and the same straight line.

At the point B, in the straight line AB, let the two straight lines BC and BD, on opposite sides of AB, make the adjacent angles ABC and ABD together = two right angles.

Then it is to be proved that

The two straight lines BC and BD are in one and the same straight line.



CONSTRUCTION.—If BC and BD are not in the same straight line, then suppose that BC and BE are, i.e. that CBE is one straight line.

PROOF.—Because CBE is a straight line, and AB is another line falling upon it in B, therefore the angles ABC and ABE together = two right angles (I. 13).

But the angles ABC and ABD together = two right angles (hyp.), and therefore the angles ABC and ABD = the angles ABC and ABE (ax. 1). Take away the common angle ABC, and then the angle ABE = the angle ABD (ax. 3), i.e. a part = the whole, which is absurd (ax. 9).

Therefore the supposition that BC and BE are in the same straight line is false.

Similarly it can be proved that only BC and BD are in the same straight line.

Therefore, it is proved, as required, that

The two straight lines BC and BD are in one and the same straight line.

Wherefore,

If at a point in a straight line, &c.

Q. E. D.

PROP. XV. THEOREM.

If two straight lines cut one another, then the vertical, or opposite, angles are equal to each other.

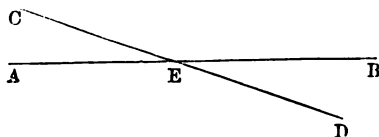
Let the two straight lines AB and CD cut each other in the point E.

Then it is to be proved that

The angle AEC = the angle BED, and

Similarly,

The angle CEB = the angle AED.



PROOF.—*Because the angles AEC and AED = two right angles, and because also the angles AED and DEB = two right angles (I. 13); therefore the angles AEC and AED = the angles AED and DEB (ax. 1); take away the common angle AED from each, then the remaining angle AEC = the remaining angle DEB.*

Therefore, it is proved, as required, that

The angle AEC = the angle BED.

Similarly,

The angle CEB = the angle AED.

Wherefore,

If two straight lines cut one another, &c.

Q. E. D.

COROLLARIES.

1. If two straight lines cut one another, the four angles they make at the point where they cut, are together equal to four right angles.

2. If any number of straight lines cut one another, all the angles made by them where they cut, are together equal to four right angles.

Exercises.

1. Prove in Prop. XV. that the angle CEB = the angle AED.

2. Prove each of the above Corollaries.

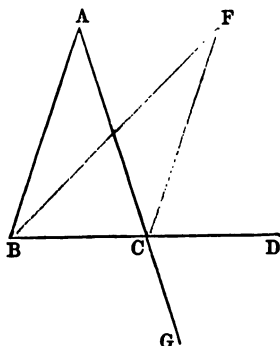
PROP. XVI. THEOREM.

If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.

Let ABC be a triangle, and let the side BC be produced to D.

Then it is to be proved that

The exterior angle ACD shall be greater than either the interior opposite angles ABC or BAC.*



CONSTRUCTION.—1. Bisect AC in E by the line BE (I. 10).

2. Produce BE to F (post. 2) making $EF = BE$ (I. 3), and join FC.

PROOF.—*Because* in the triangles AEB and FEC, we have the sides AE, and EB, and their angle AEB, in the former = the sides CE and EF, and their angle FEC, in the latter, each to each (cons. and I. 15), *therefore* the angle BAE = the angle ECF (I. 4), *i.e.* the angle BAC = the angle ACF (note 2 def. 15).

But the angle ACD is *greater* than the angle ACF (ax. 9), *i.e.* the exterior angle ACD is *greater* than the interior opposite angle BAC.

Similarly,

If we bisect BC and produce AC to G, &c., as before, *then* the angle BCG would be proved to be *greater* than the angle ABC.

But the angle $BCG =$ the angle ACD (I. 15), therefore the exterior angle ACD is *greater* than the interior opposite angle ABC .

Therefore, it is proved, as required, that

The exterior angle ACD is greater than either the int. opp. angles ABC or BAC .

Wherefore,

If one side of a triangle be produced, &c.

Q. E. D.

* With ACD as the *exterior* angle, there are three *interior* angles, BAC , ABC , and ACB . But angle ACB is *interior* and *adjacent*, whilst the angles BAC and ABC are *interior* and *opposite*; and it is of *these* the Proposition speaks.

Exercises.

1. Prove, in Prop. XVI., that when BC is bisected, and AC produced to G , the angle BCG is *greater* than the angle ABC .

2. If ABC be a triangle with the side BC produced to D , and with the exterior angle ACD *bisected* by CF , and the interior adjacent angle ACB *bisected* by CG ; prove that the angle $GCF =$ a right angle.

PROP. XVII. THEOREM.

Any two angles of a triangle are together less than two right angles.

Let ABC be a triangle.

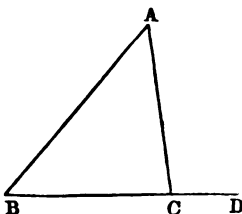
Then it is to be proved that

The angles ABC and ACB are together less than two right angles.

Similarly,

The angles ABC and BAC are together less than two right angles;

And the angles ACB and BAC are together less than two right angles.



CONSTRUCTION.—Produce the side BC to D.

PROOF.—*Because* the exterior angle ACD is *greater* than the interior and opposite angle ABC (I. 16), *add* to each of these the angle ACB, *then* the angles ACD and ACB are *greater* than the angles ABC and ACB (ax. 4).

But the angles ACD and ACB = two right angles (I. 13), and

Therefore, it is proved, as required, that

The angles ABC and ACB are *less* than two right angles.

Similarly,

The angles ABC and BAC are together less than two right angles.

And the angles ACB and BAC are together less than two right angles.

Wherefore,

Any two angles of a triangle, &c.

Q. E. D.

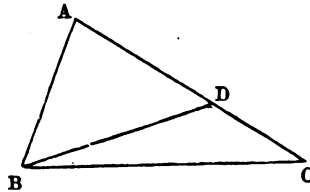
PROP. XVIII. THEOREM.

The angle which is opposite to the greater side in any triangle is greater than the angle which is opposite to the less.

Let ABC be a triangle, of which the side AC is greater than the side AB .

Then it is to be proved that

The angle ABC is greater than the angle ACB .



CONSTRUCTION.—Because AC is greater than AB (hyp.), make $AD = AB$ (I. 3), and join BD .

PROOF.—Now, the exterior angle ADB is greater than the interior and opposite angle BCD (I. 16), i.e. the angle BCA (note 2 def. 15); and the angle $ABD =$ the angle ADB (I. 5); therefore the angle ABD is also greater than the angle BCA .

Again, the angle ABC is greater than the angle ABD (ax. 9).

Therefore, it is proved, as required, that

The angle ABC is greater than the angle ACB .

Wherefore,

In any triangle, &c.

Q. E. D.

Exercise.

Prove, in Prop. XVII., that, as stated, the angles ABC and BAC are together less than two right angles.

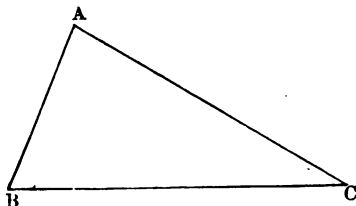
PROP. XIX. THEOREM.

The side which is opposite to the greater angle in any triangle is greater than the side which is opposite to the less.

Let ABC be a triangle, of which the angle ABC is greater than the angle ACB.

Then it is to be proved that

The side AC is greater than the side AB.



PROOF.—If the side AC is *not* greater than the side AB, then it is plain that AC must be *either* = *or* less than AB.

Now, if the side AC = AB, then the angle ABC = the angle ACB (I. 5); *but* the angle ABC is greater than the angle ACB (hyp.), *therefore* the side AC is *not* = the side AB.

Next, if the side AC is less than the side AB, then the angle ABC is less than the angle ACB (I. 18); *but* the angle ABC is greater than the angle ACB (hyp.), *therefore* the side AC is *not* less than the side AB.

Since, then, the side AC is *neither* equal to *nor* less than the side AB,

Therefore, it is proved, as required, that

The side AC is greater than the side AB.

Wherefore,

In any triangle, &c.

Q. E. D.

PROP. XX. THEOREM.

- Any two sides of a triangle are together greater than the third side.

Let ABC be a triangle.

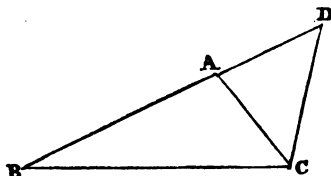
Then it is to be proved that

The sides BA and AC are together greater than the side BC.

Similarly,

The sides AB and BC are together greater than the side AC.

And the sides BC and CA are together greater than the side AB.



CONSTRUCTION.--Produce BA to D, making $AD = AC$ (I. 3), and join DC.

PROOF.—*Because* the side $AD =$ the side AC (cons.), *therefore* the angle $ADC =$ the angle ACD (I. 5); *but* the angle BCD is greater than the angle ACD (ax. 9); *therefore* the angle BCD is greater than the angle ADC , i.e. the angle BDC (note 2 def. 15), *and therefore* the side BD is greater than the side BC (I. 19).

But the side $BD =$ the sides BA and AC (cons.)

Therefore, it is proved, as required, that

The sides BA and AC are together greater than the side BC .

Similarly,

The sides AB and BC are together greater than the side AC , and

The sides BC and CA are together greater than the side AB .

Wherefore,

Any two sides of a triangle, &c.

Q. E. D.

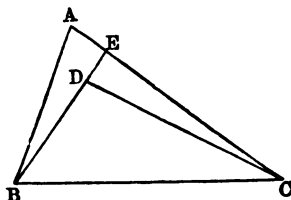
PROP. XXI. THEOREM.

If from the ends of the side of a triangle two straight lines be drawn to a point within the triangle, these shall be together less than the other two sides of the triangle, but they shall contain a greater angle.

Let ABC be a triangle, and from B and C, the ends of the side BC, let the two straight lines BD and CD be drawn to a point D within the triangle.

Then it is to be proved that

1. The two lines BD and DC are together less than the two sides BA and AC; but
2. The angle BDC is greater than the angle BAC.



CONSTRUCTION.—Produce BD to meet AC in E.

PROOF.—*Because* in the triangle BAE, the sides BA and AE are together greater than BE (I. 20), add to each of these EC; *then* the sides BA, AE, and EC, *i.e.* the sides BA and AC, are together greater than the sides BE and EC (ax. 4).

Again, *because* in the triangle CED the sides CE and ED are together greater than CD (I. 20), add to each of these DB, *then* the sides CE, ED, and DB, *i.e.* the sides CE and EB, are together greater than the sides CD and DB (ax. 4).

But we have already proved that the sides BA and AC are together greater than the sides CE and EB; *and therefore* the sides BA and AC are together much greater than the ~~sides~~ CD and BD.

Therefore, it is proved, as required, that

1. The two lines BD and DC are together less than the two sides BA and AC.

Next, *because* the exterior angle BDC of the triangle DCE is greater than the interior and opposite angle CED (I. 16), *i.e.* the angle CEB (note 2 def. 15), *and* also the exterior angle CEB is greater than the interior and opposite angle BAE (I. 16), *i.e.* the angle BAC,

Therefore, it is proved, as required, that

2. The angle BDC is greater than the angle BAC.

Wherefore,

If from the ends, &c.

Q. E. D.

Exercises.

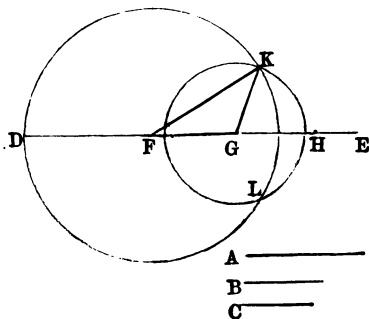
1. If from a point A two straight lines, AB and AC, be let fall upon another straight line ED, the line AB being perpendicular to ED; *prove* that AB subtends, or is opposite to, an acute angle.
2. Prove, in Prop. XX., that, as stated, the sides AB and BC are together greater than the side AC.
3. If a point A and a straight line BC be given; *prove* that the shortest line that can be drawn from A to BC, say AD, is perpendicular to BC.

PROP. XXII. PROBLEM.

To make a triangle of which the sides shall be equal to three given straight lines, any two of which are together greater than the third.

Let A, B, and C be three given lines any two of which are together greater than the third.

It is required to make a triangle having its sides = A, B, and C, each to each.



CONSTRUCTION.—1. Take a straight line DE, terminated at D, but unlimited towards E.

2. In this line make $DF = A$, make $FG = B$, and $GH = C$ (I. 3).

3. From centre F with radius FD describe the circle DKL.

4. From centre G with radius GH describe the circle HKL.

5. Join KF and KG.

Then it is to be proved that

The triangle KFG is the triangle required.

PROOF.—Because F is the centre of the circle DKL, therefore $FK = FD$ (def. 15), and $FD = A$ (cons.), therefore $FK = A$ (ax. 1).

Again, because G is the centre of the circle HKL, therefore $GK = GH$ (def. 15), and $GH = C$ (cons.), therefore $GK = C$ (ax. 1); also $FG = B$ (cons.), therefore, the triangle KFG has its three sides KF, FG, and GK = the three given lines A, B, and C, each to each.

Therefore, it is proved, as required, that

The triangle KFG is the triangle required.

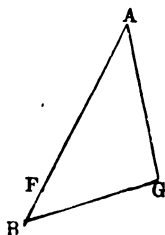
Q. E. F.

PROP. XXIII. PROBLEM.

At a given point in a given straight line, to make a rectilineal angle equal to a given rectilineal angle.

Let AB be the given straight line, A the given point in it, and DCE the given rectilineal angle.

It is required to make at A a rectilineal angle = the rectilineal angle DCE.



CONSTRUCTION.—1. In CD and CE take any points D and E and join DE.

2. Make the triangle FAG = the triangle DCE, so that the three sides FA, AG, and GF = the sides DC, CE, and ED, each to each (I. 22).

Then it is to be proved that

The angle FAG is the angle required.

PROOF.—*Because* in the triangles FAG and DCE we have the three sides FA, AG, and GF in the one = the three sides DC, CE, and ED in the other, each to each (cons.), *therefore* the angle FAG = the angle DCE (I. 8).

Therefore, it is proved that

The angle FAG is the angle required.

Q. E. F.

Exercise.

If ABC be a triangle with the side AB greater than AC; *prove* that the difference between AB and AC is less than the side BC.

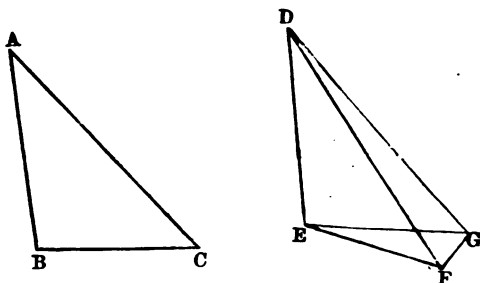
PROP. XXIV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, but the angles contained by these sides unequal, then their bases or third sides shall be unequal, and the base of that triangle which has the greater angle shall be greater than the base of the other.

Let ABC and DEF be two triangles, with the sides AB and AC in the former = the sides DE and DF , in the latter, each to each, but with the angle BAC greater than the angle EDF .

Then it is to be proved that

The base BC is greater than the base EF .



CONSTRUCTION.—Of the two sides DE and DF , let DE be not greater than DF .

1. At the point D in the straight line ED make the angle $EDG =$ the angle BAC (I. 23).

2. Make $DG = AC$ or DF , and join EG and GF .

PROOF.—*Because* in the triangles BAC and EDG we have the sides BA and AC and their angle BAC , in the former = the sides ED and DG and their angle EDG in the latter, each to each (hyp. and cons.), *therefore* the base $BC =$ the base EG (I. 4).

Again, *because* $DG = DF$ (cons.), *therefore* the angle $DGF =$ the angle DFG (I. 5).

But the angle DGF is greater than the angle EGF (ax. 9); *therefore* the angle DFG is also greater than the angle EGF .

Again, *because* the angle EFG is greater than the angle DFG (ax. 9), *therefore* the angle EFG is much greater than the angle EGF , *and therefore* the side EG is greater than the side EF (I. 19).

But it has been shown that $\text{EG} = \text{BC}$.

Therefore, it is proved, as required, that

The base BC is greater than the base EF .

Wherefore,

If two triangles, &c.

Q. E. D.

N.B.—Compare this Proposition with Prop. IV.

Exercises.

1. Prove that from the same point A *above* a given straight line CD , only one perpendicular AB can be drawn to CD .

2. Prove that from the same point A *in* a given straight line CD , only one perpendicular AB can be drawn to CD .

3. If ABC be a triangle with CD drawn from the vertical angle C , bisecting AB in D ; *prove* that AC and CB are together greater than twice CD .

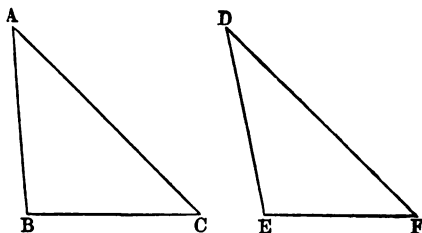
PROP. XXV. THEOREM.

If two triangles have two sides of the one equal to two sides of the other, each to each, but their bases or third sides unequal to each other, then the angles contained by the equal sides in each triangle shall be unequal, and the angle contained by the two sides of that triangle which has the greater base shall be greater than the angle contained by the two sides equal to them of the other.

Let ABC and DEF be two triangles with the sides AB and AC in the former = the sides DE and DF in the latter, each to each, but with the base BC greater than the base EF.

Then it is to be proved that

The angle BAC is greater than the angle EDF.



PROOF.—*If the angle BAC be not greater than the angle EDF, then it is plain that the angle BAC must be either =, or less than, the angle EDF.*

Now, *if the angle BAC = the angle EDF, then the base BC = the base EF (I. 4); but the base BC is greater than the base EF (hyp.); therefore the angle BAC is not equal to the angle EDF.*

Next, *if the angle BAC is less than the angle EDF, then the base BC would be less than the base EF (I. 24); but the base BC is greater than the base EF (hyp.); therefore the angle BAC is not less than the angle EDF.*

Since, then, the angle BAC is *neither equal to, nor less than*, the angle EDF,

Therefore, it is proved, as required, that

The angle BAC is greater than the angle EDF.

Wherefore,

If two triangles, &c.

Q. E. D.

N.B.—Compare this Proposition with Prop. VIII.

Exercises.

1. If ABC be a triangle, and D a point within it, with straight lines joining DA, DB, and DC; *prove* that the lines DA, DB, and DC are together *less* than the sides of the triangle AB, BC, and CA.

2. If ABCD be any quadrilateral, with AC and BD its diagonals; *prove* that the four sides AB, BC, CD, and DA are together *greater* than the diagonals AC and BD together.

PROP. XXVI. THEOREM.

If two triangles have two angles of the one equal to two angles of the other, each to each, and one side equal to one side, namely, either the sides adjacent to the equal angles in each, or the sides opposite to equal angles in each, then the other two sides shall be equal, each to each, and also the third angle of the one triangle shall be equal to the third angle of the other.

Let ABC , DEF be two triangles which have the angles ABC and ACB in the former = the angles DEF and DFE in the latter, each to each; and let the side *adjacent* to the equal angles in the former triangle = the side *adjacent* to the equal angles in the latter; or let the equal sides be those *opposite* to equal angles in each triangle.

Then

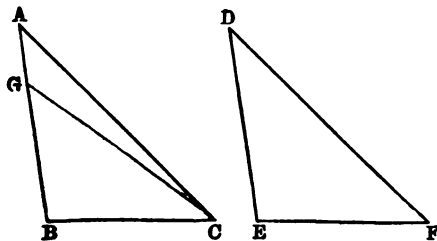
1. The other two sides in the former triangle = the other two sides in the latter; and
2. The third angle in the former triangle = the third angle in the latter.

CASE I.

First, let the given equal sides be those *adjacent* to the equal angles in each triangle, viz. BC and EF .

Then it is to be proved that

1. The other two sides AB and AC in the former triangle = the other two sides DE and DF in the latter; and
2. The third angle BAC in the former triangle = the third angle EDF in the latter.



Construction.—If AB be not equal to DE , then it is

plain that one of them must be greater than the other. Let AB be the greater; from it cut off $BG = DE$ (I. 3), and join GC .

PROOF.—*Because* in the triangles GBC and DEF we have the sides GB and BC , and their angle GBC , in the former = the sides DE and EF , and their angle DEF , in the latter, each to each (cons. and hyp.), *therefore* the angle GCB = the angle DFE (I. 4); *but* the angle DFE = the angle ACB (hyp.); *therefore* the angle GCB = the angle ACB (ax. 1), i.e. a part equals the whole, which is absurd (ax. 9). *Therefore* the supposition that AB is not equal to DE is erroneous; *consequently* $AB = DE$.

Again: *because* in the triangles ABC and DEF we now have the sides AB and BC , and their angle ABC , in the former = the sides DE and EF , and their angle DEF , in the latter, each to each (hyp.), *therefore* the base AC = the base DF , and the angle BAC = the angle EDF (I. 4).

Therefore, when the equal sides are *adjacent* to the equal angles in each triangle, it is proved, as required, that

1. The other two sides BA and AC in the former triangle = the other two sides DE and EF in the latter; and that
2. The third angle BAC in the former triangle = the third angle EDF in the latter.

Q. E. D.

Exercises.

Prove the above Case—

1. By supposing DE to be greater than AB .
2. By supposing AC to be greater than DF .
3. By supposing DF to be greater than AC .

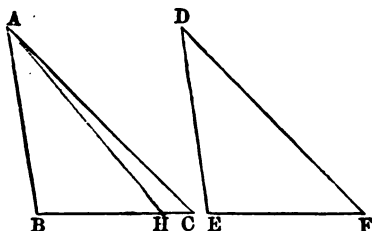
CASE II.

Secondly, let the given equal sides be those *opposite* to equal angles in each triangle, viz., AB equal to DE.

Then it is to be proved that

1. The other two sides BC and CA in the former triangle = the other two sides EF and FD in the latter; and
2. The third angle BAC of the former triangle = the third angle EDF of the latter.

CONSTRUCTION.—If BC be *not equal* to EF, *then* it is plain that one of them must be greater than the other. Let BC



be the greater; from it cut off $BH = EF$ (I. 3), and join AH.

PROOF.—*Because* in the triangles ABH and DEF we have the sides AB and BH, and their angle ABH, in the former = the sides DE and EF, and their angle

DEF, in the latter, each to each (cons. and hyp.), *therefore* the angle BHA = the angle EFD (I. 4); *but* the angle EFD = the angle BCA (hyp.); *therefore* the angle BHA = the angle BCA (ax. 1); *but* the angle BHA is *greater* than the angle HCA (I. 16), i.e. the angle BCA (note 2 def. 15). *Hence* the angle BHA is both =, and greater than, the angle BCA, which is absurd. *Therefore* the supposition that BC is *not equal* to EF is erroneous; *consequently* $BC = EF$.

Again. *Because* in the triangles ABC and DEF we now have the sides AB and BC, and their angle ABC, in the former = the sides DE and EF, and their angle DEF, in the latter, each to each, *therefore* the base AC = the base DF, and the angle BAC = the angle EDF (I. 4).

Therefore, when the equal sides are *opposite* to the equal angles in each triangle, it is proved, as required, that

1. The other two sides BC and CA in the former triangle = the other two sides EF and FD in the latter; and
2. The third angle BAC of the former triangle = the third angle DEF of the latter.

Wherefore,

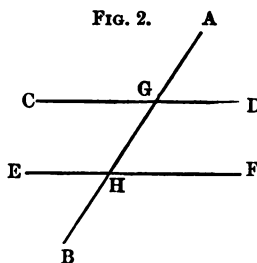
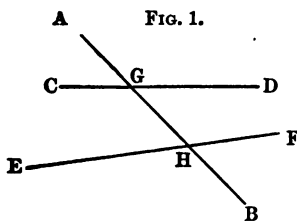
If two triangles, &c.

Q. E. D.

ANGLES MADE BY INTERSECTING LINES.

When one straight line falls upon two other straight lines, whether these two lines are parallel to each other or not, the angles formed at the several points of intersection have certain technical names; and these angles and their names must be thoroughly understood before proceeding to the next Propositions.

This is easily done by reference to the accompanying figures.



In fig. 1. Let the straight line AB cut the straight lines CD and EF, which are *not parallel* to each other, in G and H.

In fig. 2. Let the straight line AB cut the straight lines CD and EF, which are *parallel* to each other, in G and H.

Then, in *each* figure,

1. The angles AGC and AGD *above* CD are *exterior*, or outside, angles.

Also the angles BHE and BHF *below* EF are *exterior*, or outside, angles.

2. The angles CGH and DGH *below* CD are *interior*, or inside, angles.

Also the angles EHG and FHG *above* EF are *interior*, or inside, angles.

3. The angles CGH and GHF are one pair of *alternate* angles.

And the angles DGH and GHE are also a pair of *alternate* angles.

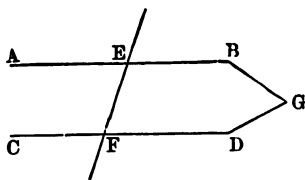
PROP. XXVII. THEOREM.

If a straight line falling upon two other straight lines make the alternate angles equal to one another, these two straight lines are parallel to each other.

Let the straight line EF falling upon the two straight lines AB and CD make the alternate angles AEF and EFD = each other.

Then it is to be proved that

The straight lines AB and CD are parallel.



CONSTRUCTION.—*If AB is not parallel to CD, then AB and CD being produced will meet, either towards A and C, or towards B and D.*

Let them be produced towards B and D, and meet in G.

PROOF.—*Because AB and CD are produced to meet in G (hyp.), therefore EGF is a triangle, and the exterior angle AEF is greater than the interior opposite angle EFG (I. 16).*

But also the angle AEF = the angle EFD (hyp.), i.e. the angle EFG (note 2 def. 15), which is absurd.

Therefore the supposition that AB and CD meet when produced towards B and D is erroneous.

Similarly, the supposition that AB and CD meet if produced towards A and C, is erroneous.

Therefore, it is proved, as required, that

The straight lines AB and CD are parallel (def. 35).

Wherefore,

If a straight line, &c.

Q. E. D.

N.B.—In the above figure the scholar must not be puzzled because EGF is called a *triangle*. The reason why it is not triangle in appearance is because the size of the page does not permit the lines AB and CD to meet to form an actual triangle.

Exercises.

1. Prove Prop. XXVI. Case ii. by taking AC and DF as the pair of equal sides *opposite* to the equal angles in each triangle.

2. If AD bisecting the vertical angle A of a triangle $\triangle ABC$ be perpendicular to the base BC; *prove* that the triangle is isosceles.

3. If A and B are two points, A above and B below a straight line CD, and CD bisects AB in E; *prove* that if AF and BG be drawn to CD at right angles to it, $AF = BG$.

PROP. XXVIII. THEOREM.

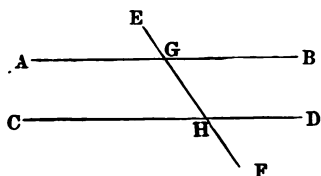
If a straight line falling upon two other straight lines make the exterior angle equal to the interior and opposite angle on the same side of the line; or make the two interior angles on the same side together equal to two right angles, these two straight lines are parallel to each other.

Let the straight line EF falling upon the two straight lines AB and CD make

1. *Either the exterior angle EGB = the interior and opposite angle GHD on the same side of EF, or*
2. *The two interior angles BGH and GHD on the same side of EF = two right angles.*

Then, in either case, it is to be proved that

The straight lines AB and CD are parallel.



PROOF.—1. *Because the angle EGB = the angle GHD (hyp.), and because the angle EGB = the angle AGH (I. 15), therefore the angle AGH = the angle GHD (ax. 1); and these are alternate angles.*

Therefore, it is proved, as first required, that

The straight lines AB and CD are parallel.

2. *Because the angles BGH and GHD = two right angles (hyp.), and because the angles AGH and BGH = two right angles (I. 13), therefore the angles AGH and BGH = the angles BGH and GHD (ax. 1); remove the common angle BGH, and then the remaining angle AGH = the remaining angle GHD (ax. 3); and these are alternate angles.*

Therefore, it is proved, as required in the second place,
that

The straight lines AB and CD are parallel.

Wherefore,

If a straight line, &c.

Q. E. D.

Exercises.

1. Prove the above Proposition by taking the angles on the left of the cutting line EF, viz. the exterior angle EGA, the interior and opposite angle GHC, and the two interior angles AGH and GHC.
2. If any angle BAC be bisected by a straight line AD, and any point E be taken in AD; prove that if straight lines EF and EG be drawn to AB and AC respectively, perpendicular to AD, then $EF = EG$.
3. If in a triangle ABC, with vertex C, the sides AC and BC be bisected at right angles by DF and EF respectively meeting in F; prove that FG drawn at right angles to the third side AB will bisect it in G.

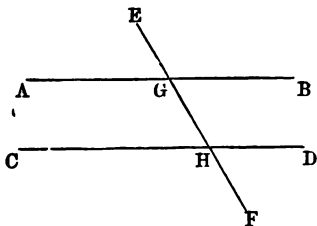
PROP. XXIX. THEOREM.

If a straight line fall on two parallel straight lines it makes the alternate angles equal to one another; the exterior angle equal to the interior and opposite angle on the same side; and also the two interior angles on the same side taken together equal to two right angles.

Let the straight line EF fall on the two parallel lines AB and CD.

Then it is to be proved that

1. The alternate angle $AGH =$ the alternate angle GHD .
2. The exterior angle $EGB =$ the interior and opposite angle GHD .
3. The two interior angles BGH and $GHD =$ two right angles.



PROOF.—1. *If the angle AGH is not equal to the angle GHD , then one of them must be greater than the other.*

Suppose that the angle AGH is greater than the angle GHD , and to each of them add the angle BGH ; then the angles AGH and BGH are greater than the angles GHD and BGH (ax. 4).

But the angles AGH and $BGH =$ two right angles (I. 13); therefore the angles BGH and GHD must be less than two right angles; and therefore the straight lines AB and CD will meet if produced (ax. 12). But they never can meet when produced, because by hypothesis they are parallel

(def. 35). *Therefore* the supposition that the angle AGH is *greater* than the angle GHD is erroneous. *Similarly*, the supposition that the angle AGH is *less* than the angle GHD might be shewn to be erroneous. *Consequently*, the angle AGH = the angle GHD.

Therefore, it is proved, as required, that

1. The alternate angle AGH = the alternate angle GHD.
2. Next: *Because* the angle EGB = the angle AGH (I. 15), *and because*, as it has just been proved, the angle AGH = the angle GHD, *therefore* the angle EGB = the angle GHD (ax. 1).

Therefore, it is proved, as required, that

2. The exterior angle EGB = the interior and opposite angle GHD.
3. Further: *Because*, as we have just proved, the angle EGB = the angle GHD, *add* to each of them the angle BGH, *then* the angles EGB and BGH = the angles BGH and GHD (ax. 2). *But* the angles EGB and BGH = two right angles (I. 13), *therefore* the angles BGH and GHD = two right angles (ax. 1).

Therefore, it is proved, as required, that

3. The two interior angles BGH and GHD = two right angles.

Wherefore,

If a straight line, &c.

Q. E. D.

Exercise.

1. Prove the above Proposition by taking
 1. The alternate angles BGH and GHD;
 2. The angles on the left of EF, viz. the exterior angle EGA, with its interior and opposite on the same side, GHD; and
 3. The two interior angles AGH and GHD.

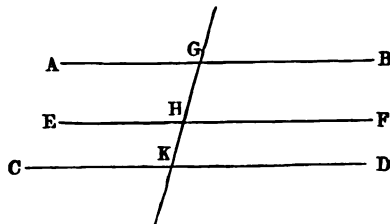
PROP. XXX. THEOREM.

Straight lines which are parallel to the same straight line are parallel to each other.

Let AB and CD be straight lines, parallel to the same straight line EF.

Then it is to be proved that

The straight lines AB and CD are parallel.



CONSTRUCTION.—Let the straight line GHK cut the straight lines AB, EF, and CD, in the points G, H, and K, respectively.

PROOF.—*Because the straight lines AB and EF are parallel (hyp.), therefore the alternate angle AGH = the alternate angle GHF (I. 29).*

Next, *because the straight lines EF and CD are parallel (hyp.), therefore the exterior angle GHF = the interior and opposite angle HKD (I. 29).*

But it has been just proved that the angle AGH = the angle GHF; therefore the angle AGH = the angle HKD, i.e. the angle AGK = the angle GKD (note 2 def. 15); and these are alternate angles.

Therefore, it is proved, as required, that

The straight lines AB and CD are parallel (I. 27).

Wherefore,

Straight lines, &c.

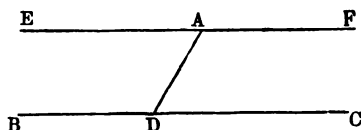
Q. E. D.

PROP. XXXI. PROBLEM.

To draw a straight line through a given point, parallel to a given straight line.

Let A be the given point, and BC the given straight line.

It is required to draw a straight line through A, which shall be parallel to BC.



CONSTRUCTION.—1. In the straight line BC, take any point, D, and join AD.

2. At the point A in the straight line AD make the angle $EAD =$ the angle ADC (I. 23), and produce the straight line EA to F.

Then it is to be proved that

The straight line EF drawn through the point A, is parallel to the straight line BC.

PROOF.—*Because* the straight line AD falling upon the two straight lines EF and BC makes the alternate angle $EAD =$ the alternate angle ADC (cons.), *therefore* EF is parallel to BC (I. 27).

Therefore, it is proved, as required, that

The straight line EF drawn through the point A is parallel to the straight line BC.

Q. E. F.

Exercise.

If from the extremities of two equal and parallel straight lines AB and CD, straight lines AD and BC are drawn, joining their extremities and intersecting in E; *prove* that $AE = ED$ and that $BE = EC$.

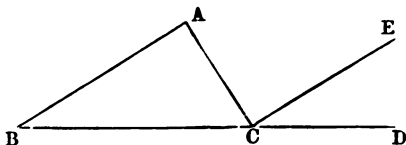
PROP. XXXII. THEOREM.

If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are together equal to two right angles.

Let ABC be a triangle, and let one of its sides, BC , be produced to D .

Then it is to be proved that

1. The exterior angle ACD = the two interior and opposite angles CAB and ABC ; and
2. The three interior angles of the triangle, ABC , BCA , and CAB = two right angles.



CONSTRUCTION.—Through the point C draw CE parallel to BA (I. 31.)

PROOF.—1. *Because* AB is parallel to CE (cons.), and AC falls upon them, *therefore* the alternate angle ACE = the alternate angle BAC (I. 29).

Also because AB is parallel to CE (cons.), and BD falls upon them, *therefore* the exterior angle ECD = the interior and opposite angle ABC (I. 29).

We have therefore the angles ACE and ECD = the angles BAC and ABC . *But* the angles ACE and ECD make together the angle ACD , and *therefore* the angle ACD = the angles BAC and ABC (ax. 2).

Therefore, it is proved, as required, that

1. The exterior angle ACD = the two interior and opposite angles, CAB and ABC .
2. Next. *Because*, as we have just proved, the angle ACD = the angles BAC and ABC , add to each of these equals the

angle ACB, and then the angles ACD and ACB = the angles CAB, ABC, and ACB (ax. 2).

But the angles ACD and ACB = two right angles I. 13), and therefore also the angles CAB, ABC, and BCA = two right angles (ax. 1).

Therefore, it is proved, as required, that

2. The three interior angles of the triangle, ABC, BCA, and CAB = two right angles.

Wherefore,

If a side of any triangle, &c.

Q. E. D.

Exercises.

1. If AD bisecting the vertical angle A of a triangle ABC, bisects also the base BC in D; *prove* that ABC is an isosceles triangle.

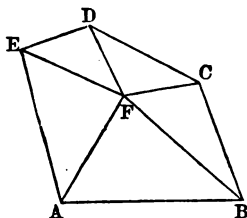
2. If ABC be a triangle right-angled at A, and AD be drawn to bisect the base BC in D; *prove* that $AD = BD$ or DC .

3. If ABC be an isosceles triangle with vertex A, and the side BA be produced beyond A to D; *prove* that the exterior angle CAD = twice the angle ABC or the angle ACB, the angles at the base.

COROLLARIES.

Corollary I

All the interior angles of any rectilineal figure, together with four right angles $\} = \left\{ \begin{array}{l} \text{Twice as many right} \\ \text{angles as the figure} \\ \text{has sides.} \end{array} \right.$



Because any rectilineal figure, as ABCDE, can, by drawing straight lines from a point F, within the figure, to each angle, be divided into *as many triangles as the figure has sides*, as is shewn in the above figure, *and because* the three interior angles of every triangle equal two right angles (I. 32),

Therefore

All the interior angles of the triangles $\} = \left\{ \begin{array}{l} \text{Twice as many right angles} \\ \text{as there are triangles.} \end{array} \right.$
 $\} = \left\{ \begin{array}{l} \text{Twice as many right angles} \\ \text{as the figure has sides.} \end{array} \right.$

But because

All the interior angles of the triangles $\} = \left\{ \begin{array}{l} \text{The interior angles of the figure} \\ \text{with the angles at F;} \end{array} \right.$

And because

The angles at F = four right angles (I. 15, Cor. 2);

Therefore

All the interior angles of the figure *with four right angles* $\} = \left\{ \begin{array}{l} \text{All the interior angles} \\ \text{of the triangles;} \end{array} \right.$
 $\} = \left\{ \begin{array}{l} \text{Twice as many right} \\ \text{angles as the figure} \\ \text{has sides.} \end{array} \right.$

Similarly it may be demonstrated for any other rectilineal figure.

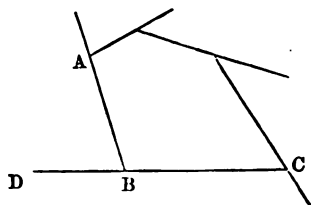
Therefore, it is proved, as required, that

All the interior, &c.

Q. E. D.

Corollary II.

All the exterior angles of any } = { Four right angles.
rectilineal figure together



Because the interior angle ABC *with* its adjacent exterior angle ABD = two right angles (I. 13),

Therefore

All the interior *with* all the exterior angles of the figure } = { Twice as many right angles as the figure has sides.

But because

All the interior angles of the figure *with* four right angles } = { Twice as many right angles as the figure has sides (I. 32, Cor. 1),

Therefore

All the interior *with* all the exterior angles of the figure } = { All the interior angles of the figure *with* four right angles (ax. 1).

And taking away from each the *interior* angles common to both,

Then

All the exterior angles of the figure = four right angles (ax. 3).

Similarly it may be demonstrated for any other rectilineal figure.

Therefore, it is proved, as required, that

All the exterior angles of any } = Four right angles.
rectilineal figure

Q. E. D.

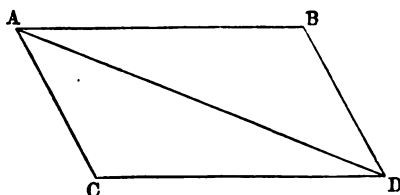
PROP. XXXIII. THEOREM.

The straight lines which join the extremities of two equal and parallel straight lines, towards the same parts, are also themselves equal and parallel.

Let AB and CD be two equal and parallel straight lines, joined towards the same parts by the straight lines AC and BD.

Then it is to be proved that

1. The straight lines AC and BD = each other.
2. The straight lines AC and BD are parallel to each other.



CONSTRUCTION.—Join AD (post. 1).

PROOF.—1. *Because* AB is parallel to CD, and AD meets them (hyp.), *therefore* the alternate angle BAD = the alternate angle ADC (I. 29).

Next, *because* in the triangles BAD and ADC, we have the sides BA and AD, and their angle BAD, in the former = the sides CD and DA, and their angle CDA, in the latter, each to each (hyp. cons. and proof above), *therefore* the angle ADB = the angle DAC, and the base AC = the base BD (I. 4).

Therefore, it is proved, as required, that

1. The straight lines AC and BD = each other.

2. Further, *because* the straight line AD meets the two straight lines AC and BD, and makes the alt. angle ADB = the alt. angle DAC, as above demonstrated,

Therefore, it is proved, as required, that

2. The straight lines AC and BD are parallel to each other.

Wherefore,

The straight lines, &c.

Q. E. D.

Exercises.

1. Prove the above Proposition by joining BC.
2. Prove Prop. XXXII. by producing the side CB beyond B to D.
3. If ABC be a triangle with vertex A and base BC; *prove* that the line DF bisecting AB and AC in D and F respectively is parallel to the base BC.

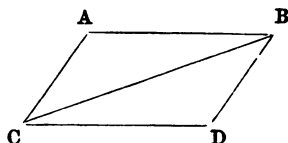
PROP. XXXIV. THEOREM.

The opposite sides and angles of a parallelogram are equal to one another, and the diameter bisects the parallelogram, that is, divides it into two equal parts.

Let $ABDC$ be a parallelogram, of which BC is a diameter.

Then it is to be proved that

1. The opposite sides and angles of the parallelogram are equal to one another, viz., $AB = CD$, and $AC = BD$, the angle $CAB =$ the angle CDB , and the angle $ABD =$ the angle ACD ; and
2. The diameter BC divides the parallelogram into two equal parts.



PROOF.—1. *Because* AB is parallel to CD (def. 34) and BC meets them, *therefore* the alternate angle $ABC =$ the alternate angle BCD (I. 29).

And because AC is parallel to BD (hyp.) and BC meets them, *therefore* the alternate angle $ACB =$ the alternate angle CBD (I. 29).

Next, *because* in the triangles ABC and BCD we have the angles ABC and BCA , and the adjacent side BC in the former $=$ the angles BCD and CBD and the adjacent side BC in the latter, each to each (cons. and I. 29), *therefore* the side $AB =$ the side CD , and the side $AC =$ the side BD , and the angle $CAB =$ the angle CDB (I. 26).

Also, because the angles ABC and $ACB =$ the angles BCD and CBD , each to each, as here proved, *i.e.* the whole angle $ABD =$ the whole angle ACD ,

Therefore, it is proved, as required, that

1. The opposite sides and angles of a parallelogram are equal to one another.
2. Next, *because* in the triangles ABC and BCD we have the sides AB and BC, and their angle ABC, in the former = the sides DC and CB, and their angle DCB, in the latter, each to each, as here proved ; *therefore* the triangle ABC = the triangle BCD (I. 4).

Therefore, it is proved, as required, that

2. The diameter divides the parallelogram into two equal parts.

Wherefore,

The opposite sides, &c.

Q. E. D.

Exercises.

1. Prove Prop. XXXIV. by joining AD.
2. If ABC be an isosceles triangle with the vertical angle A a right angle ; *prove* that each of the angles at the base, ABC and ACB = half a right angle.
3. If ABCD be a parallelogram on base AB, and AC and BD its diagonals, intersecting in E ; *prove* that AE and BE = CE and DE, each to each.

PROP. XXXV. THEOREM.

Parallelograms on the same base, and between the same parallels, are equal to each other.

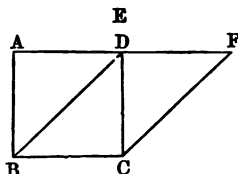
Let ABCD and EBCF be parallelograms on the same base BC, and between the same parallels AF and BC.

Then it is to be proved that

The parallelogram ABCD = the parallelogram EBCF.

This Proposition is considered under three Cases.

CASE I.—In this Case the sides AD and EF opposite to the base BC are *both terminated in point D*.

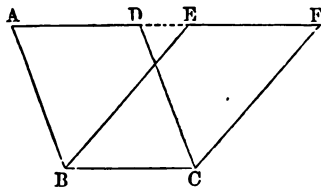


PROOF.—*Because the triangle BDC is half the parallelogram ABCD (I. 34), and because the triangle BDC is also half the parallelogram EBCF (I. 34),*

Therefore, it is proved in this Case, as required, that
(ax. 6)

The parallelogram ABCD = the parallelogram EBCF.

CASE 2.—In this Case let there be a space between the sides AD and EF opposite to the base BC.



PROOF.—*Because ABCD is a parallelogram (hyp.), there-*

fore $AD = BC$ (I. 34), and for the same reason $EF = BC$, therefore $AD = EF$ (ax. 1); then add DE to each, and the whole EA = the whole FD (ax. 2); also, $AB = DC$ (I. 34), and the exterior angle FDC = the interior and opposite angle EAB (I. 29).

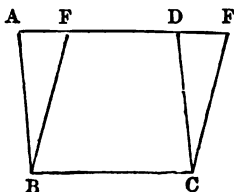
Now, because in the triangles EAB and FDC we have the sides EA and AB and their angle EAB , in the former = the sides FD and DC and their angle FDC , in the latter, each to each (as just proved), therefore the triangle EAB = the triangle FDC (I. 4).

Next, if we take the triangle EAB from the trapezium $FABC$, we have left the parallelogram $EBCF$; and if we take the triangle FDC from the same trapezium $FABC$, we have left the parallelogram $DABC$; and these remainders are equal (ax. 3).

Therefore, it is proved in this Case, as required, that

The parallelogram $ABCD$ = the parallelogram $EBCF$.

CASE 3.—In this case the sides AD and EF opposite the base BC overlap each other as in the following figure.



PROOF.—The same method of proof applies in this Case as in Case 2, the only difference being that in Case 2 we added ED to AD and EF , and here we have to take ED away from each.

Wherefore,

Parallelograms on the same base, &c.

Q. E. D.

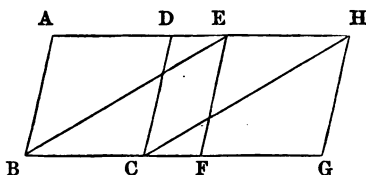
PROP. XXXVI. THEOREM.

Parallelograms on equal bases and between the same parallels are equal to one another.

Let ABCD and EFGH be parallelograms on equal bases BC and FG, and between the same parallels AH and BG.

Then it is to be proved that

The parallelogram ABCD = the parallelogram EFGH



CONSTRUCTION.—Join BE and CH.

PROOF.—Because $BC = FG$ (hyp.), and because $FG = EH$ (I. 34), therefore $BC = EH$ (ax. 1); also, because BC and EH are parallels and joined towards the same parts by BE and CH (hyp.), therefore BE and CH are both equal and parallel (I. 33); and therefore the figure EBCH is a parallelogram (def. 34).

Next, because the parallelograms ABCD and EBCH are upon the same base BC and between the same parallels AH and BC, therefore the parallelogram ABCD = the parallelogram EBCH (I. 35).

Similarly, the parallelogram EFGH = the parallelogram EBCH, being upon the same base EH, and between the same parallels EH and BG.

Therefore, it is proved, as required, that (ax. 1)

The parallelogram ABCD = the parallelogram EFGH.

Wherefore,

Parallelograms on equal bases, &c.

Q. E. D.

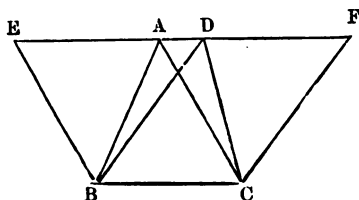
PROP. XXXVII. THEOREM.

Triangles on the same base and between the same parallels are equal to one another.

Let ABC and DBC be triangles on the same base BC, and between the same parallels AD and BC.

Then it is to be proved that

The triangle ABC = the triangle DBC.



CONSTRUCTION.—1. Produce AD both ways to points E and F.

2. Through B draw BE parallel to CA, and through C draw CF parallel to BD (I. 31).

PROOF.—*Because* each of the figures EBCA and DBCF is a parallelogram (def. 34), *and because* they are on the same base BC, and between the same parallels BC and EF, *therefore* the parallelogram EBCA = the parallelogram DBCF (I. 35).

Next, *because* the triangle ABC is half the parallelogram EBCA, *and* the triangle DBC is half the parallelogram DBCF (I. 34),

Therefore, it is proved, as required, that

The triangle ABC = the triangle DBC (ax. 7).

Wherefore,

Triangles on the same base, &c.

Q. E. D.

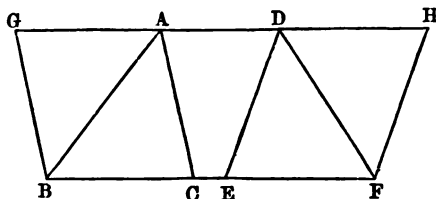
PROP. XXXVIII. THEOREM.

Triangles on equal bases and between the same parallels are equal to one another.

Let ABC and DEF be triangles on equal bases BC and EF, and between the same parallels AD and BF.

Then it is to be proved that

The triangle ABC = the triangle DEF.



CONSTRUCTION.—1. Produce AD both ways to G and H.

2. Through B draw BG parallel to CA, and through F draw FH parallel to ED (I. 31).

PROOF.—*Because each of the figures GBCA and DEFH is a parallelogram (def. 34), and because they are on equal bases BC and EF, and between the same parallels GH and BF, therefore the parallelogram GBCA = the parallelogram DEFH (I. 36).*

Next, *because the triangle ABC is half the parallelogram GBCA, and because the triangle DEF is half the parallelogram DEFH (I. 34),*

Therefore, it is proved, as required, that (ax. 7)

The triangle ABC = the triangle DEF.

Wherefore,

Triangles on equal bases, &c.

Q. E. D.

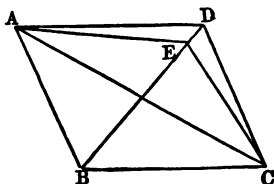
PROP. XXXIX. THEOREM.

Equal triangles on the same base, and on the same side of it are between the same parallels.

Let ABC and DBC be equal triangles, on the same base BC and on the same (viz. the upper) side of it.

Then it is to be proved that

The triangles ABC and DBC are between the same parallels.



CONSTRUCTION.—1. Draw AD joining the vertices of the triangles (post. 1); then, if AD is not parallel to their base BC ,

2. Through A draw AE parallel to this base BC , meeting BD in E (I. 31), and join EC .

PROOF.—*Because* the triangles ABC and EBC are on the same base BC (hyp.), and between the same supposed parallels AE and BC , *therefore* the triangle $ABC =$ the triangle EBC (I. 37).

But the triangle $ABC =$ the triangle DBC (hyp.), *therefore*, also, the triangle $DBC =$ the triangle EBC (ax. 1), i.e. the whole = its part, which is impossible (ax. 9).

Therefore the supposition that AE is parallel to BC is erroneous; and *similarly* it can be proved that only AD , joining the vertices of the triangles, is parallel to their base BC .

Therefore, it is proved, as required, that

The triangles ABC and DBC are between the same parallels.

Wherefore,

Equal triangles on the same base, &c.

Q. E. D.

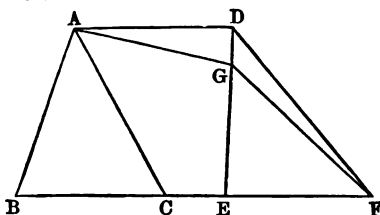
PROP. XL. THEOREM.

Equal triangles, on equal bases, in the same straight line, and on the same side of it, are between the same parallels.

Let ABC and DEF be equal triangles, on equal bases BC and EF , in the same straight line BF , and on the same (viz. the upper) side of it.

Then it is to be proved that

The triangles ABC and DEF are between the same parallels.



CONSTRUCTION.—1. Draw AD , joining the vertices of the triangles (post. 1); then, if AD is not parallel to the line of their bases BF ,

2. Through A draw AG parallel to this line BF , meeting ED in G (I. 31), and join GF .

PROOF.—Because the triangles ABC and GEF are upon equal bases BC and EF (hyp.), and between the same supposed parallels AG and BF , therefore the triangle $ABC =$ the triangle GEF (I. 38).

But the triangle $ABC =$ the triangle DEF (hyp.), therefore, also, the triangle $DEF =$ the triangle GEF (ax. 1), i.e. the whole = its part, which is impossible (ax. 9).

Therefore the supposition that AG is parallel to BF is erroneous; and similarly it can be proved that only AD , joining the vertices of the triangles, is parallel to the line of their bases, BF .

Therefore, it is proved, as required, that

The triangles ABC and DEF are between the same parallels.

Wherefore,

Equal triangles, on equal bases, &c.

Q. E. D.

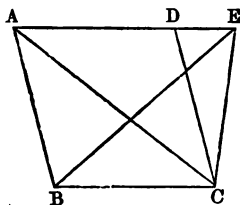
PROP. XLI. THEOREM.

If a parallelogram and a triangle be on the same base, and between the same parallels, the parallelogram shall be double of the triangle.

Let ABCD be a parallelogram, and EBC a triangle, on the same base BC, and between the same parallels BC and AE.

Then it is to be proved that

The parallelogram ABCD is double the triangle EBC.



CONSTRUCTION.—Join AC.

PROOF.—*Because* the triangles ABC and EBC are upon the same base BC, and between the same parallels BC and AE, *therefore* the triangle ABC = the triangle EBC (I. 37).

But the parallelogram ABCD is double the triangle ABC (I. 34).

Therefore, it is proved, as required, that

The parallelogram ABCD is double the triangle EBC.

Wherefore,

If a parallelogram and a triangle, &c.

Q. E. D.

Exercise.

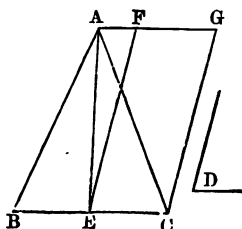
If ABCD be a quadrilateral with base AB and diagonals AC and BD, bisecting each other in E; *prove* that ABCD is a parallelogram.

PROP. XLII. PROBLEM.

To describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

Let ABC be the given triangle, and D the given rectilineal angle.

It is required to describe a parallelogram = the triangle ABC, and having one of its angles = the angle D.



CONSTRUCTION.—1. Bisect BC in E (I. 10), and join AE.

2. At the point E in the line EC, make the angle CEF = the angle D (I. 23).

3. Through A draw AG parallel to EC, and through C draw CG parallel to EF (I. 31).

Then it is to be proved that

FECG is a parallelogram = the triangle ABC, and having the angle CEF = the angle D.

PROOF.—Because the triangles ABE and AEC are upon equal bases BE and EC (cons.), and between the same parallels BC and AG (cons.), therefore the triangle ABE = the triangle AEC (I. 38), and therefore the triangle ABC is double the triangle AEC.

But because the parallelogram FECG (cons. and def. 34) and the triangle AEC are upon the same base EC, and between the same parallels EC and AG, therefore the parallelogram FECG is double the triangle AEC (I. 41).

But we have seen that the triangle ABC is double the triangle AEC, therefore the parallelogram FECG = the triangle ABC (ax. 6), and it has one of its angles CEF = the angle D (cons.).

Therefore, it is proved, as required, that

FECG is a parallelogram = the triangle ABC, and having the angle CEF = the angle D.

Q. E. F.

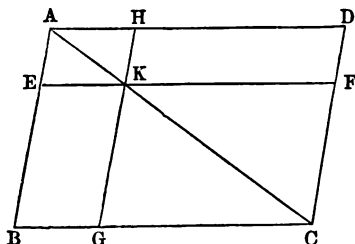
PROP. XLIII. THEOREM.

The complements of the parallelograms which are about the diameter of any parallelogram are equal to one another.

Let ABCD be a parallelogram, of which AC is a diameter, with the parallelograms AEKH and KGCF about the diameter, i.e. through which the diameter AC passes.

Then the parallelograms EBGK and HKFD, which make up the whole parallelogram ABCD, are the complements, and it is to be proved that

The complement EBGK = the complement HKFD.



PROOF.—*Because* ABCD is a parallelogram, and AC its diameter, *therefore* the triangle ABC = the triangle ADC [I. 34].

Similarly the triangle AEK = the triangle AHK, and the triangle KGC = the triangle KFC.

Hence the triangles AEK and KGC taken together = the triangles AHK and KFC taken together (ax. 2).

But we have seen that the whole triangle ABC = the whole triangle ADC; *therefore* the remainder, or the parallelogram EBGK = the remainder, or the parallelogram HKFD (ax. 3); and these are the complements in the parallelogram ABCD.

Therefore, it is proved, as required, that

The complement EBGK = the complement HKFD.

Wherefore,

The complements of the parallelograms, &c.

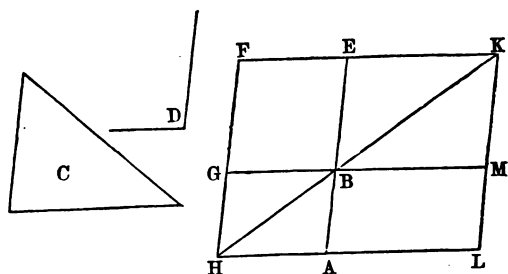
Q. E. D.

PROP. XLIV. PROBLEM.

To a given straight line to apply a parallelogram which shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

Let AB be the given straight line; C the given triangle; and D the given rectilineal angle.

It is required to apply to the straight line AB a parallelogram = the triangle C, and containing an angle = the angle D.



CONSTRUCTION (1).—1. Make the parallelogram BEFG = the triangle C, and with an angle EBG = the angle D (I. 42), so that BE may be in the same right line with AB.

2. Produce FG to H.

3. Through A draw AH parallel to BG or EF (I. 31), and join HB.

PROOF (1).—*Because the straight line HF falls on the parallel lines AH and EF, therefore the two interior angles AHF and HFE = two right angles (I. 29), and therefore the angles BHF and HFE are less than two right angles; consequently the straight lines HB and FE will meet, if produced far enough (ax. 12).*

CONSTRUCTION (2).—1. Produce the straight lines HB and FE to meet as in K.

2. Through K draw KL parallel to EA or FH (I. 31), and produce HA and GB to the points L and M.

Then it is to be proved that

ABML is a parallelogram = the triangle C, applied to the straight line AB, and having the angle ABM = the angle D.

PROOF (2).—Because FL* is a parallelogram (cons. and lef. 34) of which HK is a diameter, and FB and BL complements of the parallelograms GA and EM, about the diameter, *herfore* the parallelogram FB = the parallelogram BL (I. 13). But the parallelogram FB = the triangle C (cons.), *herfore* the parallelogram BL = the triangle C (ax. 1), and it is applied to the line AB, for it is one of its sides.

Again, because the angle GBE = the angle D (cons.), and because the angle ABM = the angle GBE (I. 15), *therefore* the angle ABM = the angle D (ax. 1).

Therefore, it is proved, as required, that

ABML is a parallelogram = the triangle C, applied to the straight line AB, and having the angle ABM = the angle D.

Q. E. F.

* N.B.—Parallelograms may be referred to by the two letters standing at *opposite* angles, instead of the four, at each angle. Thus the parallelogram FHLK may be referred to as the parallelogram FL, or KH.

Exercises.

1. If ABCD be a parallelogram with diagonal AC = diagonal BD; *prove* that each angle of the parallelogram A, B, C, and D is a right angle.

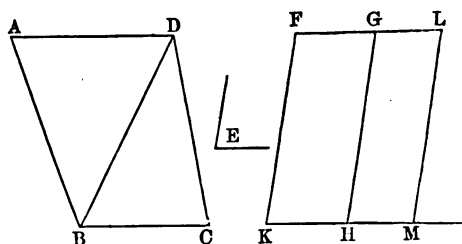
2. If ABCD be a parallelogram with its diagonals AC and BD intersecting at right angles in E; *prove* that the sides of the parallelogram AB, BC, CD, and DA = each other.

PROP. XLV. PROBLEM.

To describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given rectilineal angle.

Let $ABCD$ be the given rectilineal figure, and E the given rectilineal angle.

It is required to describe a parallelogram = the figure $ABCD$, and having an angle = the angle E .



CONSTRUCTION.—1. Join BD and describe the parallelogram FH = the triangle ABD , and having the angle FKH = the angle E (I. 42).

2. Apply to the straight line GH the parallelogram GM equal to the triangle DBC , and having the angle GHM = the angle E (I. 44).

Then it is to be proved that

FM is a parallelogram = the figure $ABCD$, and having the angle FKM = the angle E .

PROOF.—*Because each of the angles FKH and GHM = the angle E (cons.), therefore the angle FKH = the angle GHM (ax. 1); add to each the angle GKH , then the angles FKH and GKH = the angles GHM and GKH (ax. 2).*

But the angles FKH and GKH = two right angles (I. 29); therefore the angles GHM and GKH = two right

angles, and therefore KH is in the same straight line with IM (I. 14).

2. Next, *because* the lines FG and KM are parallel (cons.), and GH meets them, *therefore* the alternate angle FGH = the alternate angle GHM (I. 29); *add to each* the angle LGH, *then* the angles FGH and LGH = the angles GHM and LGH (ax. 2).

But the angles MHG and LGH = two right angles (I. 29); *therefore* the angles HGF and LGH = two right angles, *and therefore* FG is in the same right line with GL (I. 14).

3. Next, *because* KF is parallel to HG, and HG to ML (cons.), *therefore* KF is parallel to ML (I. 30), *and* KM and FL are also parallel (cons.); *therefore* the figure FKML is a parallelogram (def. 34).

Again, *because* the parallelogram FH = the triangle ABD (cons.), *and* the parallelogram GM = the triangle DBC (cons.), *therefore* the parallelogram FM = the figure ABCD, *and* it has the angle FKM = the angle E (cons.)

Therefore, it is proved, as required, that

FM is a parallelogram = the figure ABCD, and having an angle FKM = the angle E.

Q. E. F.

Corollary.—A parallelogram may be described equal to a given rectilineal figure, of any number of sides, and having one of its angles equal to a given angle.

N.B.—The learning of this Proposition will be simplified if it be observed—

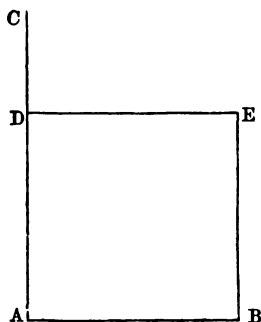
1. That the given figure is divided into triangles.
2. That a parallelogram is made equal to each triangle, the first according to Euclid I. 42, the second, and more if there be any, according to I. 44.
3. That the first part of the proof is to shew that KH and HM are in one and the same straight line.
4. That the second part is to shew that FG and GL are in one and the same straight line.

PROP. XLVI. PROBLEM.

To describe a square upon a given straight line.

Let AB be the given straight line.

It is required to describe a square upon AB.



CONSTRUCTION.—1. From A draw AC at right angles to and greater than AB (I. 11), cutting off $AD = AB$ (I. 3).

2. Through D draw DE parallel to AB: and through B draw BE parallel to AD (I. 31).

Then it is to be proved that

The figure ABED is a square upon AB.

PROOF.—1. The figure ABED is equilateral.

Because DE is parallel to AB (cons.), *and* BE is parallel to AD (cons.), *therefore* the figure ABED is a parallelogram (def. 34), *and therefore, also,* $AB = DE$ and $AD = BE$ (I. 34).

But also $AB = AD$ (cons.), *therefore* the four sides AB, BE, ED, and DA = each other, *and therefore* the figure ABED is equilateral, which was first to be shewn.

2. The figure ABED is rectangular.

Next, *because* the lines AB and DE are parallel (cons.), *and* the line AD falls upon them, *therefore* the two interior angles BAD and ADE = two right angles (I. 29).

But *because* BAD is a right angle (cons.), *therefore* ADE a right angle (ax. 3); *and because* the opposite angles of parallelograms are equal to each other (I. 34), *therefore* the angles BED and ABE are also right angles, *and therefore* each of the angles of the figure = a right angle, *and* the figure ABED is rectangular. Which was next to be shown.

Hence, the figure ABED, being equilateral and rectangular is a square (def. 30), and it is described upon AB (cons.).

Therefore, it is proved, as required, that

The figure ABED is a square upon AB.

Q. E. F.

Exercises.

1. On a given straight line MN construct a square, with perp. NP drawn from N.
2. About MO, as a diagonal, construct a square MNOP.

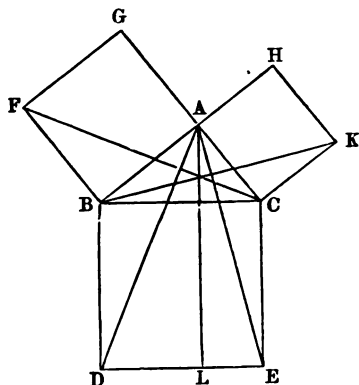
PROP. XLVII. THEOREM.

In any right-angled triangle, the square which is described upon the side subtending the right angle equals the sum of the squares described upon the sides containing the right angle.

Let ABC be a right-angled triangle having the right angle BAC .

Then it is to be proved that

The square described upon BC , the side *subtending*—i.e. *opposite to*—the right angle BAC = the sum of the squares described upon BA and AC , the sides *containing* that angle.



CONSTRUCTION.—1. On BC describe the square $BDEC$: on BA describe the square $BAGF$: and on AC describe the square $AHKC$ (I. 46).

2. Through A draw AL parallel to BD or CE (I. 31), and join AD and FC .

PROOF.—*Because* the adjacent angles BAC and BAG = two right angles (hyp. and cons.), *therefore* CAG is one and the same straight line (I. 14).

And, for the same reason, BAH is one and the same straight line.

Next, because the angles DBC and FBA are each a right angle (cons.), therefore the angle DBC = the angle FBA; add to each the angle ABC, and therefore the angle DBA = the angle FBC (ax. 2).

And because in the triangles ABD and FBC we have the sides AB and BD and their angle ABD in the former = the sides FB and BC and their angle FBC in the latter, each to each, therefore the triangle ABD = the triangle FBC (I. 4).

Again, because the parallelogram BL and the triangle ABD are upon the same base BD, and between the same parallels AL and BD, therefore the parallelogram BL is double the triangle ABD (I. 41). And, for the same reason, the square GB is double the triangle FBC. But, as we have proved, the triangle ABD = the triangle FBC; therefore the parallelogram BL = the square GB (ax. 6).

In the same manner, by joining AE and BK, it is proved that the parallelogram CL = the square HC, and therefore the whole square BDEC = the sum of the squares GB and HC (ax. 2). But BDEC is the square on BC, subtending the right angle BAC, and GB and HC are the squares on BA and AC, the sides containing the right angle BAC;

Therefore, it is proved, as required, that

The square described on the side BC subtending the right angle = the sum of the squares described on the sides BA and AC, containing that angle.

Wherefore,

In any right-angled triangle, &c.

Q. E. D.

Exercises.

1. Prove, as stated in the above proposition, that the parallelogram CL = the square HC.

2. Prove that if in the figure of Prop. 47 the points E and C are joined by the line EK, and the points D and F by the line DF, the triangles KCE and FBD are equal to each other.

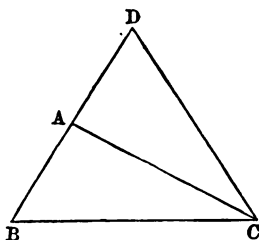
PROP. XLVIII. THEOREM.

If the square described upon one of the sides of a triangle be equal to the squares described upon the other two sides of it, the angle contained by these two sides is a right angle.

Let ABC be a triangle, with the square described upon BC = the sum of the squares described upon BA and AC.

Then it is to be proved that

The angle BAC is a right angle.



CONSTRUCTION.—From A draw AD at right angles to AC (I. 11), make AD equal to AB (I. 3), and join DC.

PROOF.—*Because* AD = AB (cons.), *therefore* the square on AD = the square on AB; *and if* the square on AC be added to each of these equals, *then* the squares on AD and AC = the squares on AB and AC (ax. 2).

And because the angle DAC is a right angle (cons.), *therefore* the square on DC = the squares on AD and AC (I. 47).

Also the square on BC = the squares on AB and AC (hyp.); *therefore* the square on DC = the square on BC (ax. 1), and *therefore* DC = BC.

Next, *because* in the triangles BAC and DAC we have the sides BA, AC, and CB in the former = the sides DA,

C, and CD in the latter, each to each (cons. and proof above), *therefore* the angle BAC = the angle DAC (I. 8), and *cause* the angle DAC is a right angle (cons.),

Therefore, it is proved, as required, that,

The angle BAC is a right angle.

Wherefore,

If the square, &c.

Q. E. D.

ADDENDUM.

BOOK I.

CLASSIFICATION OF PROPOSITIONS IN BOOK I.

I.

Straight Lines.

Props. 2, 3, 10, 11, 12, 14.

II.

Angles in connection with Straight Lines.

Props. 9, 13, 15, 23.

III.

Parallel Lines.

Props. 27, 28, 30, 31, 33.

IV.

Angles in connection with Parallel Lines.

Prop. 29.

V.

Construction of Triangles.

Props. 1, 7, 22.

VI.

Angles in connection with Triangles.

Props. 5, 16, 17, 18, 21, 32, 48.

VII.

Sides in connection with Triangles.

Props. 6, 19, 20, 47.

VIII.

Comparison of Triangles.

Props. 4 and 24, 8 and 25, 26.

IX.

Triangles and Parallel Lines.

Props. 37, 38, 39, 40.

X.

Triangles and Parallelograms.

Prop. 41

XI.

Properties and Comparison of Parallelograms.

Props. 34, 35, 36, 42, 43, 44, 45.

XII.

Construction of the Square.

Prop. 46.

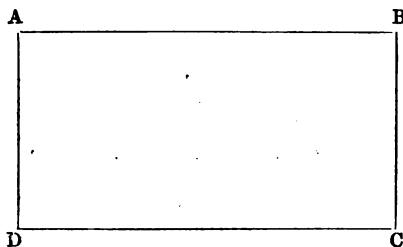
BOOK II.



DEFINITIONS.

I.

A Rectangle, *or* right-angled parallelogram, is said to be contained by any two of the straight lines which contain one of the right angles.



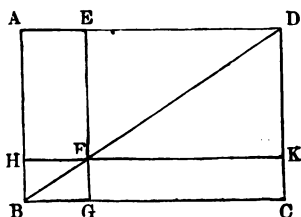
1. The rectangle ABCD is said to be contained by AB and BC, *or* by BC and CD, *or* by CD and DA, *or* by DA and AB.

2. The expression 'the rectangle AB, BC,' is allowed to be used instead of the larger one 'the rectangle contained by AB and BC.'

3. The rectangle is often referred to by the two letters standing at its opposite angles, as, in the above, rect. AC, *or* rect. DB, *or* by AC *or* BD only.

II.

very parallelogram the figure composed of *either* of parallelograms about the diameter, together with the two complements, is called a *gnomon*.



the parallelogram ABCD, the parallelogram EK about the diameter, together with the complements AF and FC, forms the *gnomon* AKG.

Similarly the parallelogram HG, about the diameter, together with the complements AF and FC, forms the *gnomon* EHC.

The *gnomon* is briefly expressed by the letters at the opposite corners of the parallelograms composing it. Thus the first *gnomon* AKG, composed of EK, AF, and FC, may be termed the *gnomon* AKG or HEC. Also the second *gnomon* EHC, composed of HG, AF, and FC, may be termed the *gnomon* EHC or AGK.

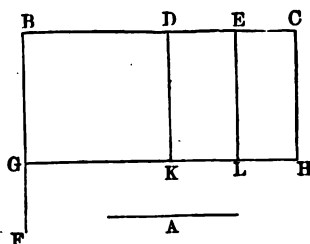
PROP. I. THEOREM.

If there be two straight lines, one of which is divided into any number of parts, the rectangle contained by the two straight lines is equal to the rectangles contained by the undivided line and the several parts of the divided line.

Let A and BC be two straight lines, one of which, BC, is divided into parts in D and E.

Then it is to be proved that

The rectangle contained by
the two straight lines A
and BC $\left\{ \begin{array}{l} \text{The rectangles contained by} \\ \text{A and BD, by A and DE,} \\ \text{and by A and EC.} \end{array} \right. =$



CONSTRUCTION.—1. From B draw BF at right angles to BC (I. 11), and cut off BG = A (I. 3).

2. Through G draw GH parallel to BC, meeting CH drawn parallel to BG (I. 31).

3. Through D and E draw DK and EL parallel to BG or CH (I. 31).

PROOF.—1. It is evident that the figure BH = the sum of the figures BK, DL, and EH.

2. But because BH is contained by BG and BC, of which BG = A (cons.), therefore BH is the rectangle contained by A and BC, the two given straight lines.

3. *Similarly* BK, DL, and EH are respectively the rectangles contained by BG and BD, by DK and DE, and by L and EC—i.e. the rectangles contained by A and BD, by A and DE, and by A and EC (cons. and I. 34).

Therefore, it is proved, as required, that

The rectangle contained by the two straight lines A and BC = the rectangles contained by A and BD, by A and DE, and by A and EC.

Wherefore,

If there be two straight lines, &c.

Q. E. D.

Exercise.

Prove the above Proposition with MN and O, the given straight lines, MN being divided into parts in P, Q, and R.

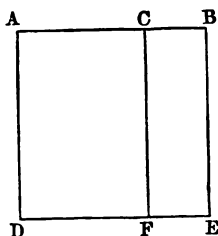
PROP. II. THEOREM.

If a straight line be divided into any two parts, the rectangles contained by the whole line and each of its parts are together equal to the square on the whole line.

Let AB be a straight line divided into any two parts in C.

Then it is to be proved that

The rectangles contained by AB and AC and by AB and BC together } = { The square on AB.



CONSTRUCTION.¹—1. On AB describe the square ADEB (I. 46).

2. Through C draw CF parallel to AD or BE (I. 31).

PROOF.—1. It is evident that the figure AE = the sum of the figures AF and CE.

2. But *because* AE is the square on AB (cons.), and *because* AF and CE are respectively the rectangles AD, AC, and CF, CB, i.e. the rectangles AB, AC, and AB, CB (cons., and I. 34),

Therefore, it is proved, as required, that

The rectangles contained by AB and AC and by AB and BC together = the square on AB.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

¹ It may be taken as a rule in constructing the figures ops. II.–VIII. inclusive, in this Book, that the first thing to be done is to construct the square spoken of in the Enunciation, and if more than one square is mentioned, the larger to be constructed.

Exercise.

If MN be a given straight line divided into any two parts in O, prove that

The rectangles contained by MN and MO, and by MN and NO, together = the square on MN.

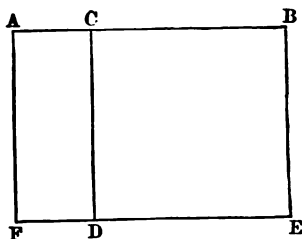
PROP. III. THEOREM.

If a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square on the aforesaid part.

Let AB be a straight line divided into any two parts in C.

Then it is to be proved that

The rectangle $\left. \begin{array}{l} \text{AB, BC} \end{array} \right\} = \left\{ \begin{array}{l} \text{The rectangle AC, CB, together with} \\ \text{the square on BC.} \end{array} \right.$



CONSTRUCTION.—1. On BC describe the square CDEB (I. 46).

2. Produce ED to F, meeting AF drawn parallel to CD or BE (I. 31).

PROOF.—1. It is evident that the figure AE = the sum of the figures AD and CE.

2. But *because* AE is contained by AB and AF, of which $AF = CD$ (I. 34) = BC (cons.), *therefore* AE is the rectangle AB, BC.

3. Next, *because* AD is the rectangle contained by AC, AF, of which $AF = BC$ (as before), *therefore* AD is the rectangle AC, CB.

4. Also CE is the square on BC (cons.).

Therefore, it is proved, as required, that

The rectangle AB, BC = the rectangle AC, CB, together with the square on BC.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Exercise.

Prove the other case of the above Proposition :

The rectangle AB, AC = the rectangle AC, CB, together with the square on AC.

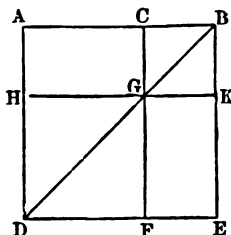
PROP. IV. THEOREM.

If a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the parts.

Let AB be a straight line, divided into any two parts in C.

Then it is to be proved that

The square on AB = { The squares on AC and CB together
with twice the rectangle AC, CB.



CONSTRUCTION.¹—1. On AB describe the square ADEB (I. 46) and join BD.

2. Through C draw CF parallel to AD or BE (I. 31) cutting BD in G.

3. Through G draw HGK parallel to AB or DE.

PROOF.²—1. *Because* CF and AD are parallel, and BD falls upon them (cons.), *therefore* ext. angle BGC = int. opp. angle GDA (I. 29), *i.e.* the angle BDA (note def. 15).

Next, *because* AB = AD (cons.), *therefore* the angle ABD = the angle ADB (I. 5), *i.e.* the angle CBG = the angle CGB (ax. 1), and *therefore* CB = CG (I. 6); and *since* CB and CG = GK and BK respectively (I. 34), *therefore* CB, CG, GK, and KB = each other (ax. 1), and *therefore* the figure CK is equilateral.

2. Further, *because* CG and BK are parallels, and CB falls upon them (cons.), *therefore* the two int. angles GCB and CBK = two right angles (I. 29). But *because* CBK is a right angle (cons.) *therefore* GCB is a right angle (ax. 1); and *since* the angles CBK and GCB = the angles CGK and GKB respectively (I. 34), *therefore* the angles CBK, GCB, CGK,

nd GKB each = a right angle, and therefore the figure CK is rectangular. It has also been proved to be equilateral, and therefore CK is a square (def. 30), and it is described on the side CB.

3. Similarly the figure HF is a square, on the side HG = AC (I. 34).

4. Next, because the comp. AG = the comp. GE (I. 43),
and also = the rect. AC, CG,
= the rect. AC, CB (cons.),
therefore the comp. GE = the rect. AC, CB (ax. 1);
and therefore the comps. AG and GE = twice the rect. AC, CB.

5. Now, it is evident that $\left. \begin{array}{l} \text{the whole figure ADEB} \\ \text{the whole figure ADEB} \end{array} \right\} = \left\{ \begin{array}{l} \text{the sum of the figures} \\ \text{AG, GE, HF, and CK.} \end{array} \right.$

But, as above, AG and GE = twice the rectangle AC, CB;
Also HF and CK = the squares on AC and CB respectively;
And the whole figure ADEB = the square on AB (cons.);

Therefore, it is proved, as required, that

The square on AB = the squares on AC and CB, together with twice the rectangle AC, CB.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Cor.—The parallelograms about the diameter of a square are squares.

¹ In addition to the note as to Construction, Prop. I., it may now be stated that the figures of Prop. IV.–VIII. require, after the square has been described, the drawing of the diagonal, and of the parallel, or parallels, employed.

² It will assist the learner to notice the following steps in this Proposition:—

- To prove that CK is the square on CB (as in 1 and 2).
- To observe that HF is the square on HG, i.e. on AC (as in 3).
- To prove that the complements AG and GE = twice rect. AC, CB (as in 4).
- To combine these conclusions in the proof of the Proposition itself (as in 5).

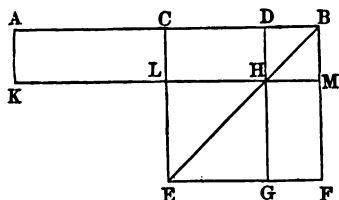
PROP. V. THEOREM.

If a straight line be divided into two equal parts, and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

Let AB be a straight line divided into two equal parts in C, and into two unequal parts in D.

Then it is to be proved that

The rectangle AD, DB together }
with the square on CD } = The square on CB.



CONSTRUCTION.—1. On CB describe the square CEFB (I. 46), and join BE.

2. Through D draw DHG parallel to CE or BF (I. 31), cutting BE in H.

3. Through H draw KLHM parallel to AB or EF, cutting CE in L and BF in M.

4. Through A draw AK parallel to CL or BM.

PROOF.—*Because* the comp. CH = the comp. HF (I. 43),

Therefore CH and DM = HF and DM (ax. 2),
i.e. the whole CM = the whole DF.

But *because* CM = AL (I. 36),

therefore AL = DF (ax. 1),

and therefore AL and CH = DF and CH (ax. 2.),
i.e. the rect. AH = the gnomon CMG.

But *because* AH is the rect. AD , DH , and $DH = DB$ [I. 4, Cor.], *therefore* $AH =$ the rect. AD , DB ,
and therefore the gnomon $CMG =$ the rect. AD , DB (ax. 1);
therefore also the gnomon CMG
 together with LG , i.e. the whole figure $CEFB$ } = { the rect. AD , DB ,
 together with LG ;
but because LG is the square on LH (II.4, Cor.) and $LH = CD$ (I. 34),

therefore the whole figure $CEFB =$ { the rect. AD , DB , together
 with the square on CD .

But the whole figure $CEFB =$ the square on CB (cons.),

Therefore, it is proved, as required, that

The rectangle AD , DB , together with the square on $CD =$ the square on CB .

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Exercises.

1. Prove that as stated in Prop. IV. Proof 3, the figure IF is a square on the side HG .

2. Prove that in Prop. V. the rectangle AD , DB , together with the square on $CD =$ the square on AC .

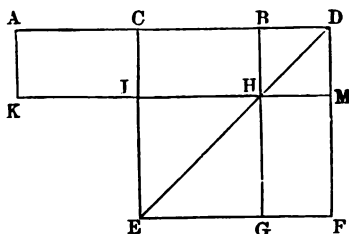
PROP. VI. THEOREM.

If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

Let AB be a straight line bisected in C, and produced to any point D.

Then it is to be proved that

The rectangle AD, DB, together
with the square on BC } = The square on CD.



CONSTRUCTION.—1. On CD describe the square CEFD (I. 46), and join DE.

2. Through B draw BG parallel to CE or DF (I. 31), cutting DE in H.

3. Through H draw KM parallel to AD or EF, cutting CE in L and DF in M.

4. Through A draw AK parallel to CL or DM.

PROOF.—

Because AL = CH (I. 36),

and CH = HF (I. 43),

therefore AL = HF (ax. 1);

and therefore AL and CM = HF and CM,

i.e. AM = the gnomon CMG.

But *because* AM is the rect. AD, DM, and DM = DB [I. 4, Cor.], *therefore* AM = rect. AD, DB,

and therefore the gnomon CMG = the rect. AD, DB (ax. 1),

therefore also the gnomon CMG together with LG, } = { the rect. AD, DB together with LG.
i.e. the whole figure CEFD

And *because* LG is the square on LH (II. 4, Cor.), and H = CB (I. 34),

therefore the whole figure CEFD = { the rect. AD, DB, together with the square on CB.

But the whole figure CEFD = the square on CD (cons.).

Therefore, it is proved, as required, that

The rectangle AD, DB together with the square on BC
= the square on CD.

Wherefore,

If a straight line be bisected, &c.

Q. E. D.

Exercise.

Prove that if in the above Proposition we take AB as bisected in C, and produced *beyond* A to D, then we should have,

The rectangle BD, AD together with the square on AC = the square on CD.

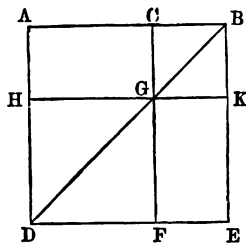
PROP. VII. THEOREM.

If a straight line be divided into any two parts, the squares on the whole line, and on one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

Let AB be a straight line divided into any two parts in C.

Then it is to be proved that

The squares on AB and BC $\left\{ = \right\}$ Twice the rectangle AB, BC, together with the square on AC.



CONSTRUCTION.—1. On AB describe the square ADEB (I. 46), and join BD.

2. Through C draw CF parallel to AD or BE (I. 31), cutting BD in G.

3. Through G draw HK parallel to AB or DE.

PROOF.— Because AG = GE (I. 43),
therefore AG and CK = GE and CK (ax. 2),
i.e. AK = CE;

and therefore AK and CE = twice AK.

But AK and CE = the gnomon AKF and CK,
therefore the gnomon AKF and CK = twice AK (ax. 1).

Next, *because* AK is the rect. AB, BK, and BK = BC
 . 4, Cor.), *therefore* AK = the rect. AB, BC,

l therefore the gnomon AKF } = { twice the rect. AB, BC
 and CK } (ax. 1);

efore also the gnomon AKF } = { twice the rect. AB, BC
 with CK and HF } and HF.

But *because* HF is the square on HG, and HG = AC (I. 34),

efore the gnomon AKF with } = { twice the rect. AB, BC,
 CK and HF } together with the
 square on AC.

t the gnomon AKF together } = { the whole figure
 with CK and HF } ADEB with CK;

nd the whole figure ADEB } = { the squares on AB and
 together with CK } BC; (cons. and II.
 4, Cor.)

Therefore, it is proved, as required, that

The squares on AB and BC = twice the rectangle AB,
 BC, together with the square on AC.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

Exercise.

Prove the other case in the above Proposition, that

The squares on AB and AC = twice the rectangle AB,
 C, together with the square on BC.

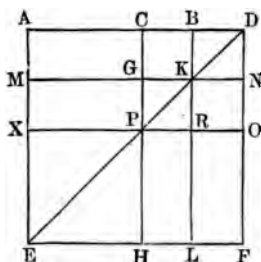
PROP. VIII. THEOREM.

If a straight line be divided into any two parts, four times the rectangle contained by the whole line and one of the parts, together with the square on the other part, is equal to the square on the straight line which is made up of the whole line and that part.

Let AB be a straight line divided into any two parts in C.

Then it is to be proved that

Four times the rectangle AB, BC, together with the square on AC = $\left\{ \begin{array}{l} \text{the square on AD,} \\ \text{i.e. on AB and} \\ \text{BC together.} \end{array} \right.$



CONSTRUCTION.—1. Produce AB to D, making BD = BC (post. 2 and I. 3).

2. Upon AD describe the square AEFD (I. 46).

3. Complete the figure by the parallels cutting the diagonal, each way, in K and P (I. 31).

PROOF.—1. *Because* BD = BC (cons.), and = KN (I. 34), *therefore* BC and BD each = KN (I. 34), *therefore* GK = KN (ax. 1), *and similarly* PR = RO.

2. Next, *because* CB = BD and GK = KN, *therefore* the rect. CK = the rect. BN, and the rect. GR = the rect. KO (I. 36).

3. But *because* the rect. CK = the rect. KO (I. 43), *therefore* the rect. BN = the rect. GR, *and therefore* the four rectangles CK, BN, GR, and KO = each other, *therefore also* they are, together, four times any one of them, as CK.

4. Next, *because* BD = BC (cons.), *and because* BD = BK (II. 4, Cor.) = CG (I. 34), *therefore* BC = CG (ax. 1); *and because* BC = GK (I. 34) = GP (II. 4, Cor.), *therefore* CG = GP (ax. 1); *and therefore* the rect. AG = the rect. MP (I. 36);

l because $PR = RO$, therefore the rect. $PL =$ the rect. RF (36). But because the rect. $MP =$ the rect. PL (I. 43) therefore these four rectangles AG , MP , PL , and $RF =$ each other, and therefore they are together four times any one of them, as AG .

5. And because the four rectangles CK , BN , GR , and $KO =$ { four times the rect. CK ;

therefore the eight rectangles $\}$ = four times the rect. AK .
making the gnomon AOH

d because AK is the rect. AB , BC , since $BK = BD$,
therefore four times the rect. AB , $BC =$ four times the rect. AK ,
= the gnomon AOH ,

l therefore four times the rect. $\}$ = { the gnomon AOH
 AB , BC , with XH with XH ,

and because $XH =$ the square on $XP = AC$,
therefore four times the rect. AB , $\}$ = { the gnomon AOH with
 BC , with the square on AC the square on AC ,
= the whole figure $A E F D$,
= the square on AD (cons.),
= { the square on AB and
 BC together.

Therefore, it is proved, as required, that

Four times the rectangle AB , BC , together with the square on $AC =$ the square on AD , i.e. on AB and BC together.

Wherefore,

If a straight line be divided, &c.

Q. E. D.

N.B.—It will assist the learner to notice the following in this Proposition :—

- To prove that the four rectangles CK , BN , GR , and $KO =$ each other, and that together they are four times any one of them, as CK (as in 1, 2, 3).
- To prove that the four rectangles AG , MP , PL , and $RF =$ each other, and that together they are four times any one of them, as AG (as in 4).
- To combine these conclusions is the proof of the Proposition itself (as in 5).

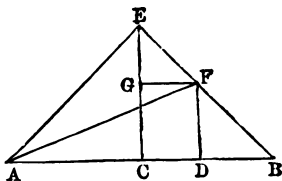
PROP. IX. THEOREM.

If a straight line be divided into two equal and also into two unequal parts, the squares on the two unequal parts are together double of the square on half the line and of the square on the line between the points of section.

Let AB be a straight line divided into two equal parts in C, and into two unequal parts in D.

Then it is to be proved that

$$\left. \begin{array}{l} \text{The squares on AD} \\ \text{and DB together} \end{array} \right\} = \left\{ \begin{array}{l} \text{double the squares on AC} \\ \text{and CD.} \end{array} \right.$$



CONSTRUCTION.—1. From C draw CE at right angles to AB and = AC or CB (I. 11, and 3).

2. Join EA and EB.

3. Through D draw DF parallel to CE (I. 31), meeting EB in F.

4. Through F draw FG parallel to AB (I. 31), meeting CE in G, and join AF.

PROOF.—1. Because AC = CE (cons.), therefore the angle CEA = the angle CAE (I. 5); and because ACE is a right

angle (cons.), *therefore* the angles CEA and CAE are together a right angle (I. 32); *and since* they are equal to each other, *therefore* each angle CEA and CAE = half a right angle.

Similarly, each angle CEB and CBE = half a right angle, *and therefore* the angles AEC and CEB together, *i.e.* the angle AEB = a right angle.

2. Next, because in the triangle EGF the angle GEF = the angle CEB (note 2 def. 15) = half a right angle, *and because* the angle EGF is a right angle, *since* the angle EGF = the angle GCB (I. 29) = the angle ECB (note def. 15) = a right angle (cons.), *therefore* the remaining angle EFG = half a right angle (I. 32), *and therefore* the side EG = the side GF (I. 6).

3. Again, because in the triangle FDB the angle FBD = the angle EBC, as above = half a right angle; *and because* the angle FDB is a right angle (cons.); *therefore* the remaining angle BFD = half a right angle = the angle FBD, *and therefore* the side DF = the side DB (I. 6).

4. Because in the triangle AEC the side AC = the side CE (cons.), *therefore* the square on AC = the square on CE, *and therefore* the squares on AC and CE together = double the square on AC.

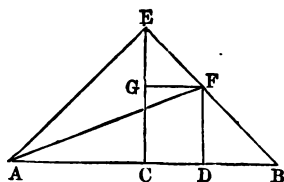
But because the square on AE = the squares on AC and CE (I. 47), *therefore* the square on AE = double the square on AC (ax. 1).

5. Because in the triangle GEF it has been proved that the side EG = the side GF, *therefore* the square on EG = the square on GF, *and therefore* the squares on EG and GF = double the square on GF.

But because the square on EF = the squares on EG and GF (I. 47), *therefore* the square on EF = double the square on GF.

And because GF = CD (I. 34), *therefore* the square on EF = double the square on CD (ax. 1).

And because, as already proved, the square on EA = double the square on AC , therefore the squares on EA and EF = double the squares on AC and CD (ax. 2).



6. Next, because in the triangle AEF the square on AF = the squares on EA and EF (I. 47), therefore the square on AF = double the squares on AC and CD (ax. 1).

7. But because in the triangle ADF the square on AF = the squares on AD and DF (I. 47), therefore the squares on AD and DF = double the squares on AC and CD (ax. 1); and since, as already proved, $DF = DB$,

Therefore, it is proved, as required, that

The squares on AD and DB together = double the squares on AC and CD .

Wherefore,

If a straight line be divided, &c.

Q. E. D.

N.B.—It will assist the learner to note the following steps in this Proposition :

- a. To prove that AEB is a right angle (as in 1).
- b. To prove that the side EG = the side GF (as in 2).
- c. To prove that the side DF = the side DB (as in 3).
- d. In the triangle AEC to prove that the square on AE = double the square on AC (as in 4).

- e. In the triangle GEF to prove that the square on EF = double the square on CD (as in 5); and to prove that the squares on AE and EF = double the squares on AC and CD.
- f. In triangle AEF to prove that the square on AF = double the squares on AC and CD (as in 6).
- g. In the triangle ADF to prove that the squares on AD and DB = double the squares on AC and CD (as in 7).

Exercises.

Prove the other case in Proposition VIII. that if AB be produced to D , making $AD = AC$, four times the rectangle contained by AB , AC , together with the square on BC = the square on BD , *i.e.* on AB and BC together.

Prove the other case in Prop. IX. that if AB be produced unequally between A and C , the squares on AD and DB together = double the squares on BC and CD .

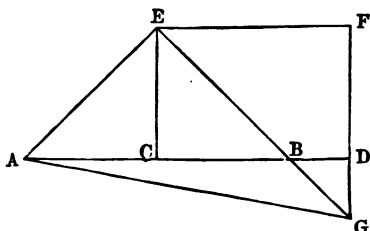
PROP. X. THEOREM.

If a straight line be bisected and produced to any point, the square on the whole line thus produced, and the square on the part of it produced, are together double of the square on half the line bisected and of the square on the line made up of the half and the part produced.

Let AB be a straight line bisected in C, and produced to D.

Then it is to be proved that

The squares on AD and DB = double the squares on AC and CD.



CONSTRUCTION (1).—1. Draw CE at right angles to AB (I. 11), and = AC or BC (I. 3).

2. Join AE and EB.

3. Draw EF parallel to CB, meeting DF drawn parallel to CE (I. 31).

Then

Because EF meets the parallels EC and FD, therefore the two int. angles CEF and EFD = two right angles (I. 29); and therefore the angles BEF and EFD are less than two right angles; and therefore EB

and FD will meet, if produced towards B and D (ax. 12).

CONSTRUCTION (2).—Let EB and FD be produced meeting in G, and join AG.

PROOF.—1. *Because* $AC = CE$ (cons.), *therefore* the angle CEA = the angle CAE (I. 5); *and because* ACE is a right angle, *therefore* the angles CEA and CAE are together a right angle (I. 32); *and since* they are equal to each other, *therefore* each angle CEA and CAE = half a right angle.

Similarly, each angle CEB and CBE = half a right angle, *and therefore* the angles CEA and CEB together, viz. the angle AEB, i.e. the angle AEG = a right angle.

2. *Because* in the triangle BDG the angle DBG = the angle CBE (I. 15) = half a right angle, as just proved, *and because* the angle BDG = the angle ECB (I. 29) = a right angle, *therefore* the remaining angle DGB = half a right angle = the angle DBG, *and therefore* the side BD = the side DG (I. 6).

3. *Because* in the triangle EGF, the angle EGF = the angle BGD = half a right angle, as just proved, *and because* the angle EFG or the angle EFD = the angle ECD (I. 34) = a right angle, *therefore* the remaining angle GEF = half a right angle = the angle EGF, *and therefore* the side FE = the side FG (I. 6).

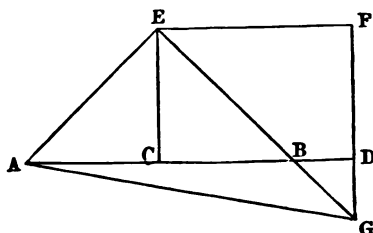
4. Again, *because* in the triangle EGF, the side FE = the side FG, as just proved, *therefore* the square on FG = the square on FE, *and therefore* the squares on FG and FE = double the square on FE.

But *because* the square on EG = the squares on FG and FE (I. 47), *therefore* the square on EG = double the square on EF (ax. 1); *and since* $EF = CD$ (I. 34), *therefore* the square on EG = double the square on CD.

5. Next, *because* in the triangle ACE, the side AC = the side CE (cons.), *therefore* the square on AC = the square on CE, *and therefore* the squares on AC and CE = double the square on AC (ax. 1).

But *because* the square on AE = the squares on AC and

CE (I. 47), *therefore the square on AE = double the square on AC* (ax. 1).



6. Now, *because the square* } = { *double the square on*
on AE } { *AC,*

and because the square on EG = { *double the square on*
CD, as above,

therefore the squares on AE and } = { *double the squares on*
EG } { *AC and CD (ax. 2).*

But the square on AG = { *the squares on AE and*
EG (I. 47),

therefore the square on AG = { *double the squares on*
AC and CD (ax. 1).

Again, the square on AG = { *the squares on AD and*
DG (I. 47),

therefore, also, the squares on } = { *double the squares on*
AD and DG } { *AC and CD;*

and since DG = *DB, as already proved,*

Therefore, it is proved, as required, that

The squares on AD and DB = double the squares on AC and CD.

Wherefore,

If a straight line be bisected, &c.

Q. E. D.

N.B.—It will assist the learner to note the following steps in this Proposition :—

- a. To prove that $\angle AEB$, *i.e.* $\angle AEG$, is a right angle (as in 1).
- b. To prove, in the triangle BDG , that $BD = DG$ (as in 2).
- c. To prove, in the triangle EGF , that $FE = FG$ (as in 3).
- d. To prove, also in the triangle EGF , that the square on $EG =$ double the square on EF , *i.e.* on CD (as in 4).
- e. To prove in the triangle ACE that the square on $AE =$ double the square on AC (as in 5).
- f. To prove from the triangles AEG and ADG , in connection with previous results, the assertion made in the Proposition.

Exercise.

Prove the other case in Prop. X. that if BA be produced to D , beyond A ,

The squares on BD and $AD =$ double the squares on BC and CD .

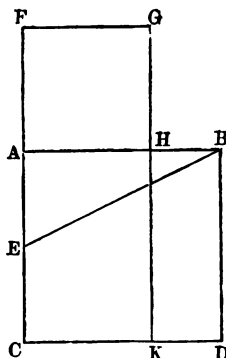
PROP. XI. PROBLEM.

To divide a straight line into two parts, so that the rectangle contained by the whole and one of the parts shall be equal to the square on the other part.

Let AB be a straight line.

It is required to divide AB into two parts, say in H, so that

The rectangle AB, BH = the square on AH.



CONSTRUCTION.—1. On AB describe the square ACDB (I. 46).

2. Bisect AC in E and join BE (I. 10, and post. 1).

3. Produce CA to F, making $EF = EB$ (post. 2, and I. 3).

4. On AF describe the square AFGH.

5. Produce GH to K, in CD.

Then it is to be proved that AB is divided in H, so that
The rectangle AB, BH = the square on AH.

PROOF.—The rect. CF, FA, } = { the square on EF
 with the square on EA } = { (II. 6).
 = { the square on EB, since
 EF = EB (cons.).

But *because* EAB is a right
 angle (cons.), *therefore* the } = { the squares on EA and
 square on EB } = { AB (I. 47),

and therefore the rect. CF, FA, } = { the squares on EA and
 with the square on EA } = { AB;

therefore also the rect. CF, FA = the square on AB (ax. 3).

But FK is the rect. CF, FA, since FG = FA and
 AD is the square on AB (cons.),

therefore FK = AD.

Take away the common part
 AK, *then* the remainder FH } = the remainder HD.

Now, FH is the square on AH (cons.), and HD is the
 rectangle DB, BH, *i.e.* AB, BH (cons.).

Therefore, it is proved, as required, that

The rectangle AB, BH = the square on AH; and the
 straight line AB is divided into two parts in H, as
 required.

Q. E. F.

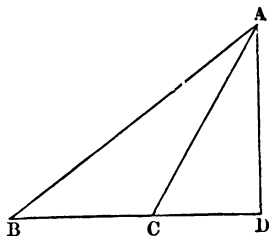
PROP. XII. THEOREM.

In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

Let ABC be an obtuse-angled triangle, having the obtuse angle ACB, and, from the acute angle A, let AD be drawn perpendicular to BC, produced to D.

Then it is to be proved that

The square on AB is *greater* than the squares on BC and CA by twice the rect. BC, CD.



PROOF.—*Because the square on BD = the squares on BC and CD, and twice the rect. BC, CD (II. 4), therefore the squares on BD and DA = the squares on BC, CD, and DA, and twice the rect. BC, CD (ax. 2).*

But because BDA is a right angle (hyp.), therefore the square on AB = the squares on BD and DA (I. 47).

Similarly, the square on CA = the squares on CD and DA.

Therefore, substituting these values (in line 2), the square on AB = the squares on BC and CA, with twice the rect. BC, CD.

Therefore, it is proved, as required, that

The square on AB is *greater* than the squares on BC and CA by twice the rectangle BC, CD.

Wherefore,

In obtuse-angled triangles, &c.

Q. E. D.

Exercise.

Prove the other case in Prop. XII. that if the perpendicular BD be drawn from the other acute angle, B, to the side AC produced to D,

The square on AB is *greater* than the squares on BC and CA by twice the rectangle AC, CD.

PROP. XIII. THEOREM.

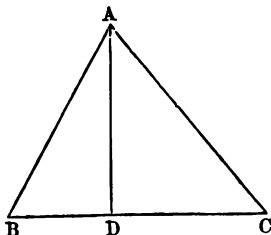
In every triangle the square on the side subtending an acute angle is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle.

Let ABC be any triangle, and the angle at B an acute angle; and on BC , one of the sides containing it, let fall the perpendicular AD from the opposite angle.

Then it is to be proved that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB , DB .

CASE I.—When the perpendicular falls *within* the triangle ABC .



PROOF.—*Because* the squares on CB and DB = twice the rect. CB , DB , with the square on DC (II. 7),

Therefore the squares on CB , DB , and DA = twice the rect. CB , DB , with the squares on DC and DA .

But because BDA is a right angle (hyp.),

Therefore the square on AB = the squares on DB and DA (I. 47).

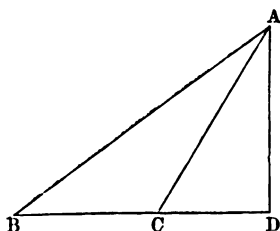
Similarly, the square on AC = the squares on AD and DC .

Therefore, substituting these values (in line 2), the squares on CB and AB = twice the rect. CB, DB with the square on AC .

Therefore, it is proved, as required in this Case, that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB, DB .

CASE II.—When the perpendicular falls *without* the triangle ABC .



PROOF.—Because the angle at D is a right angle (hyp.), and because the angle ACB is greater than the angle at D (I. 16), therefore the angle ACB is an *obtuse* angle;

and therefore the square on AB = { the squares on AC and CB , with twice the rect. CB, CD (II. 12);

therefore, also, the squares on AC and BC } = { the squares on AC, CB , BC , with twice the rect. BC, CD ,

= { the square on AC ,
twice the square on BC ,
with twice the rect. BC, CD .

But *because* the rect. BD, BC = $\left\{ \begin{array}{l} \text{the rect. BC, CD, with} \\ \text{the square on BC (II. 3),} \end{array} \right.$
therefore twice the rect. BD, BC = $\left\{ \begin{array}{l} \text{twice the rect. BC, CD,} \\ \text{with twice the square} \\ \text{on BC;} \end{array} \right.$

therefore, also, substituting these values (in lines 4, &c.), the squares on AB and BC = the square on AC, with twice the rect. BD, BC.

Therefore, it is proved, as required in this Case, that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB, DB.

CASE III.—When the perpendicular is a *side* of the triangle ABC.



PROOF.—In this Case the side BC is the straight line between the perpendicular let fall from the opposite angle A, and the acute angle taken at B.

Now, *because* the square on AB = the squares on AC and CB (I. 47), *therefore* the squares on AB and BC = the square on AC, with twice the square on CB (ax. 2).

Therefore, it is proved, as required in this Case, that

The square on AC is *less* than the squares on CB and BA by twice the rectangle CB, CB, *i.e.* CB, CD, in other Cases.

Wherefore,

In every triangle the square, &c.

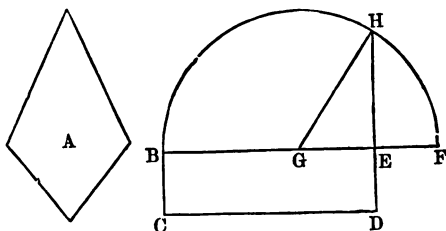
Q. E. D.

PROP. XIV. PROBLEM.

To describe a square that shall be equal to a given rectilineal figure.

Let A be the given rectilinear figure.

It is required to describe a square = the rectilinear figure A.



CONSTRUCTION.—1. Describe the rectangular parallelogram BCDE = A (I. 45), and if the sides BE and ED = each other, it is a square, and what was required is done.

2. But *if* BE and ED are *not* equal, *then* produce one of them as BE, to F, making EF = ED (post. 2, and I. 3).

3. Bisect BF in G (I. 10).

4. From centre G, with radius GB or GF, describe semi-circle BHF (post. 3).

5. Produce DE to H, and join GH (posts. 2 and 1).

Then it is to be proved that

The square described on EH = the rectilineal figure A.

PROOF.—*Because* the rect.BE,
EF, with the square on GE } = { the square on GF (II.
5),
= { the square on GH, for
GH = GF (cons.),

use the square on GH } = { the squares on GE and
 EH,
 the rect. BE, EF, with } = { the squares on GE and
 uare on GE } = { EH (ax. 1),
 efore the rect. BE, EF = the square on EH (ax. 3).
 the rect. BE, EF, is the rect. BE, ED, for EF = ED ;
 the rect. BD = the square on EH.
 the rect. BD = the figure A (cons.).

efore, it is proved, as required, that

the square described on EH = the rectilineal figure A.

Q. E. F.

ADDENDUM.

EUCLID, BOOK II.

The chief difficulty the learner has in remembering, as well as in learning, the Propositions of Euclid, Book II., arises from the verbal similarity in many of the Enunciations.

This difficulty may be lessened, in preparing for an examination at least, by taking the following Propositions in the classes referred to.

A.

Properties of a straight line divided into any two parts.
Props. 2, 3, 4, 7, and 8.



Let AB be a straight line divided into any two parts in C.

Then, Prop. 2,

The rectangles contained
by the whole and each of
the parts } = the square on the whole line.

Prop. 3,

The rectangle contained
by the whole and one part } = { the rectangle contained by
two parts, with square on
the aforesaid part.

Prop. 4,

The square on the whole
line } = { the squares on the two parts
with twice the rectangle
contained by the parts.

Prop. 7,

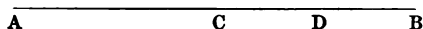
The squares on the whole line and one of the parts } = { twice the rectangle contained by the whole and that part, together with the square on the other part.

Prop. 8,

Four times the rectangle contained by the whole and one part, together with the square on the other part, } = { the square on the straight line made up of the whole and that part.

B.

Properties of a straight line divided equally and unequally.
Props. 5 and 9.



Let AB be a straight line divided equally in C and unequally in D.

Then, Prop. 5,

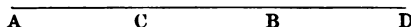
The rectangle contained by the unequal parts, together with the square on the line between the points of section, } = the square on half the line.

Prop. 9,

The squares on the unequal parts } = { double the square on half the line, together with double the square on the line between points of section.

C.

Properties of a straight line bisected and produced.
 Props. 6 and 10.



Let AB be a straight line bisected in C and produced to D.

Then, Prop. 6,

The rectangle contained
 by the whole produced line
 and the part produced, to-
 gether with the square on
 half the line bisected

$$= \left\{ \begin{array}{l} \text{the square on the straight} \\ \text{line made up of the half} \\ \text{and the part produced.} \end{array} \right.$$

Prop. 10,

The squares on the
 whole produced line and on
 the part produced

$$= \left\{ \begin{array}{l} \text{double the square on half} \\ \text{the line bisected, together} \\ \text{with double the square on} \\ \text{the line made up of the} \\ \text{half and the part produced.} \end{array} \right.$$

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